Cyclic Communication and Inseparability in MIMO Relay Channels

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joint work with Anas Chaaban

Institute of Digital Communication Systems
RUB, Bochum

Outline

1. Motivation: From one-way to multi-way
2. The MIMO Y-channel: From single-antenna to multiple-antennas
3. Main result: From capacity to DoF
4. Insights and ingredients
   - Channel diagonalization: Separability of communication structure
   - Alignment, Compute-and-forward
   - Transmission strategy(3-users)
5. Extensions and Conclusion

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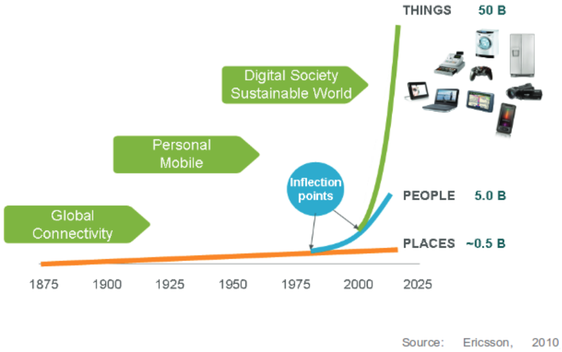
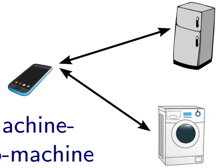
Towards 50B devices in 2020!



Increasing number of connected devices (IoT, M2M, etc.)

Towards 50B devices in 2020!

More sophisticated network topologies!



Machine-
to-machine
Point↔multi-point

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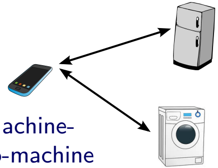

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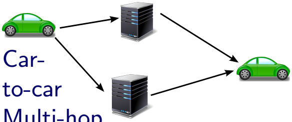

Car-
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Multi-hop

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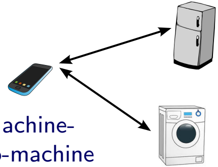

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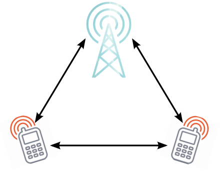
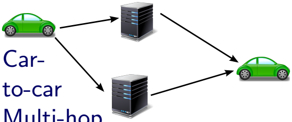


Device-to-device



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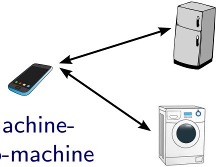

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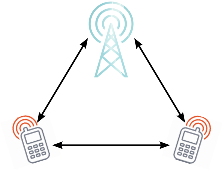

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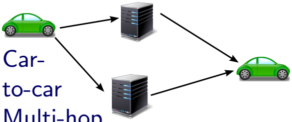

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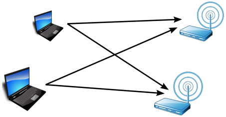

X-Channel

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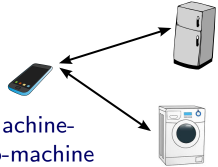

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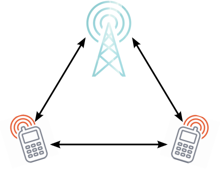

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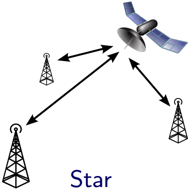
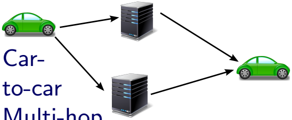


Star



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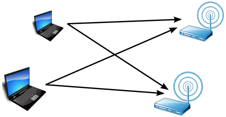

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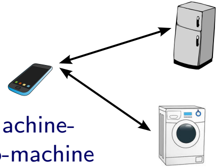

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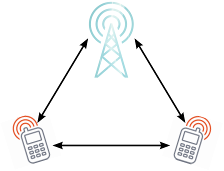

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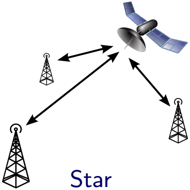

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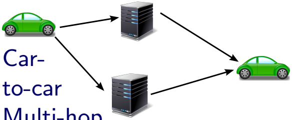

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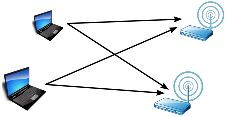

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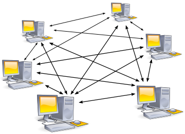

Interference channel

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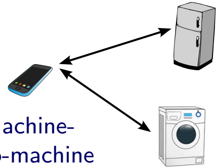

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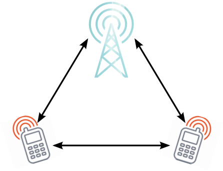

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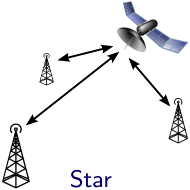

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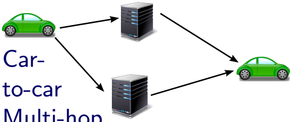

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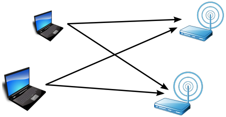

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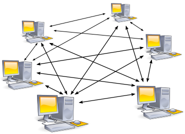

Interference channel

- Important factor: Relaying!

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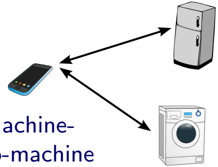

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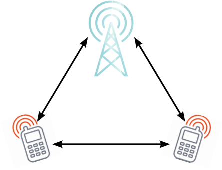

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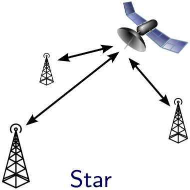

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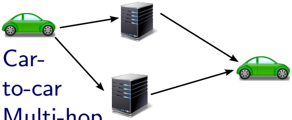

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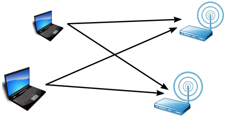
Multi-hop



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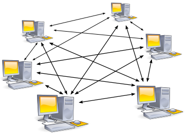

Interference channel

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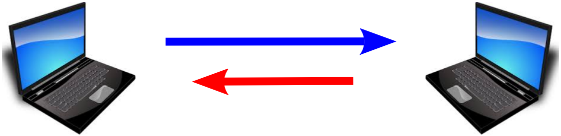
Question

Is communication mainly one-way?

Multi-way Communications

- Part of our daily communication is uni-directional

Multi-way Communications



- Part of our daily communication is uni-directional
- A large share is bi-directional (video conferencing e.g.)
  $\Rightarrow$ Two-way channel

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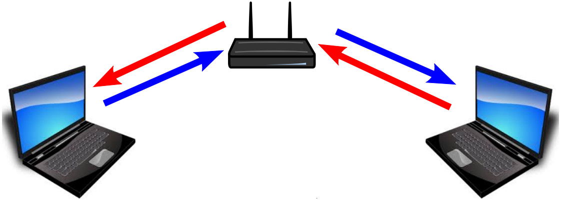

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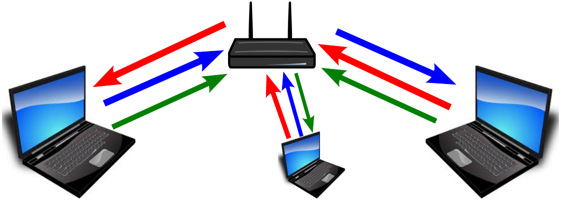
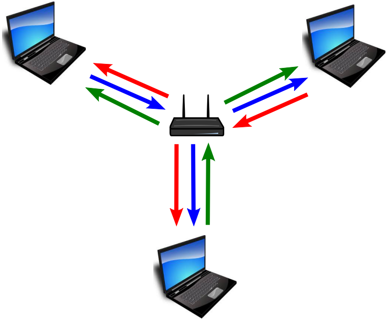
- Distant nodes
  $\Rightarrow$ Two-way relay channel

Multi-way Communications



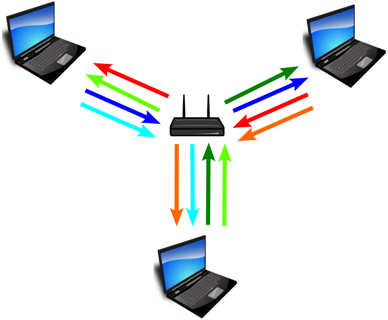
- Part of our daily communication is uni-directional
- A large share is bi-directional (video conferencing e.g.)
  $\Rightarrow$ Two-way channel
- Distant nodes
  $\Rightarrow$ Two-way relay channel
- More than 2 nodes$\Rightarrow$ Multi-way relay channel (MRC).

Multi-way Relaying



- Multi-cast MRC: a message has multiple destinations

Multi-way Relaying



- Multi-cast MRC: a message has multiple destinations
- Uni-cast MRC (Y-channel): a message has one destination

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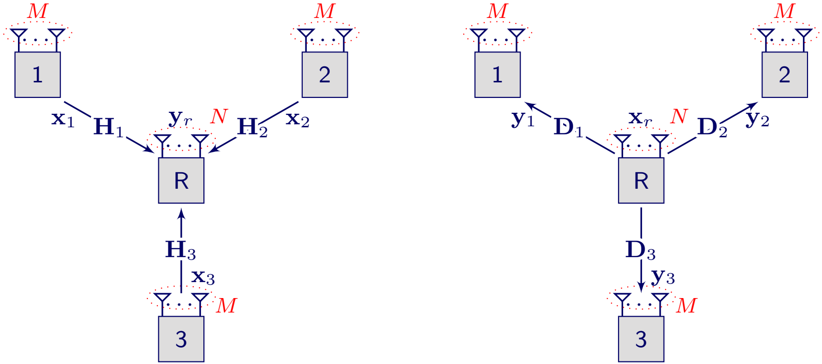
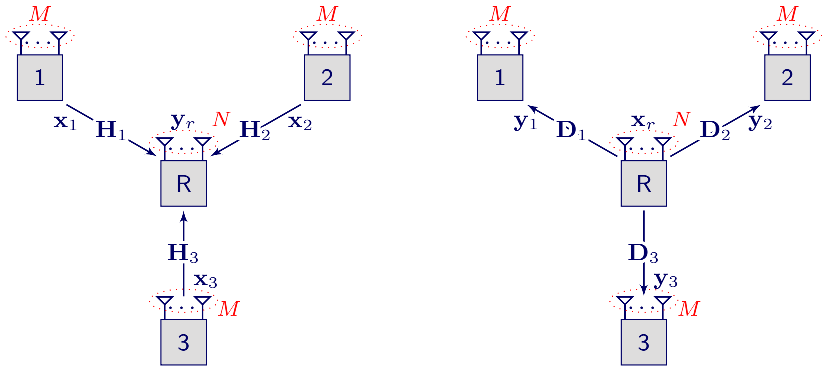
MIMO Y-channel



Uplink:

- Tx signal: $\mathbf{x}_i \in \mathbb{C}^M$, power P

MIMO Y-channel



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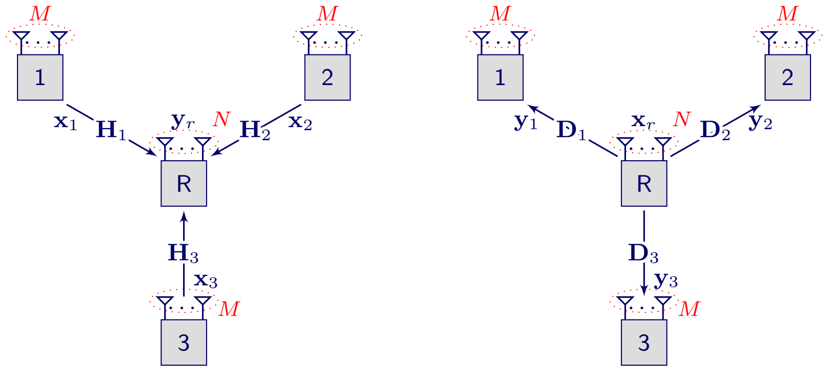

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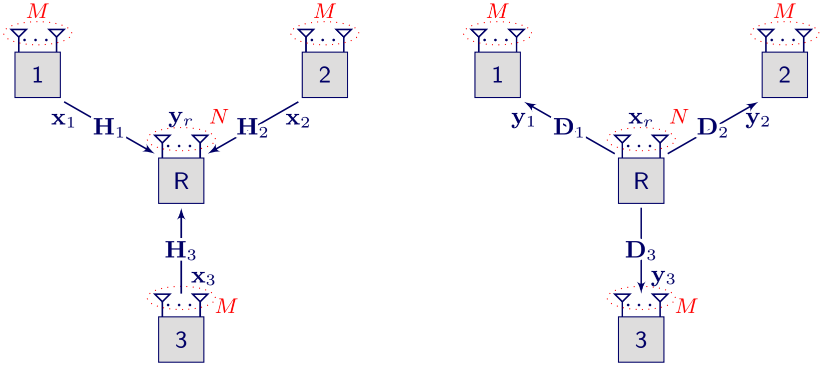

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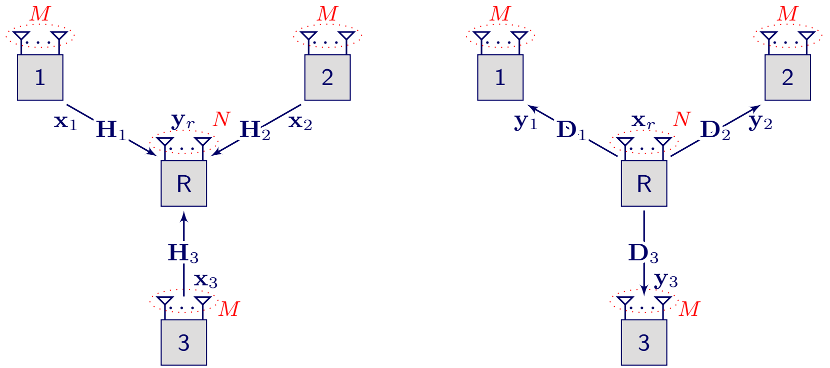

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Capacity to Capacity Region

Single-user (MIMO P2P):

Input covariance $\mathbf{Q}$, $\text{tr}(\mathbf{Q}) \leq P$

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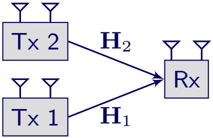

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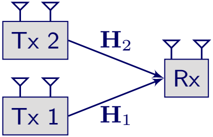

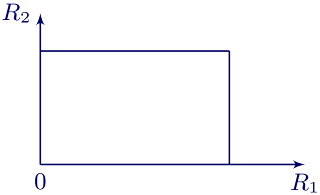

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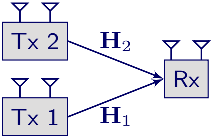

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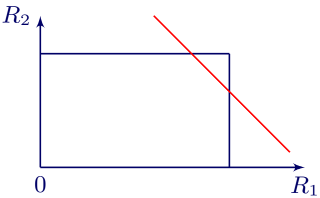

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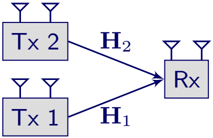

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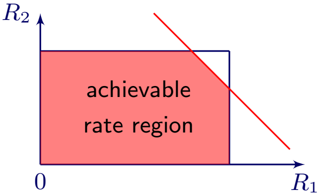

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$\Rightarrow$ $\mathbf{H} \in \mathbb{C}^{N \times M} \Rightarrow \text{rank}(\mathbf{H}) = \min\{M, N\}$

$\Rightarrow$ $C \approx \min\{M, N\} \log(P)$ at high P

Capacity to DoF

Single-user (MIMO P2P):

Capacity:

$$C = \log |\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H|$$

Optimization: water-filling

DoF:

- M Tx antennas, N Rx antennas

$\Rightarrow$ $\mathbf{H} \in \mathbb{C}^{N \times M} \Rightarrow \text{rank}(\mathbf{H}) = \min\{M, N\}$

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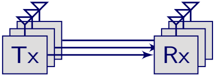
$\Rightarrow$ Capacity equivalent to that of d parallel SISO P2P channels!



(more on that later)

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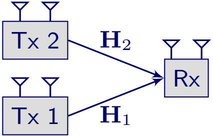

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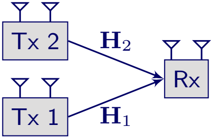

Capacity region:

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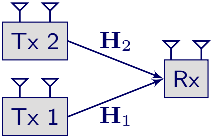

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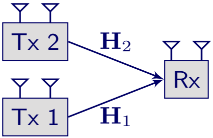

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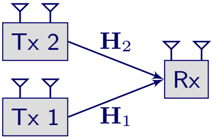

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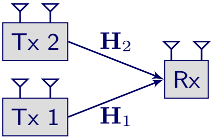

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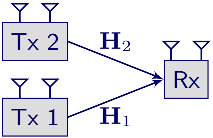

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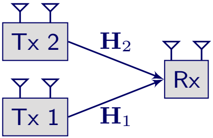

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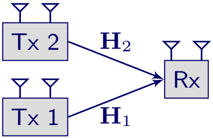

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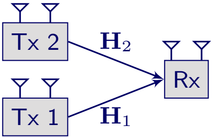

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$\Rightarrow$ Sum-capacity equivalent to that of $d_1 + d_2$ parallel SISO P2P channels!

Back to the Y-channel

Definition

R_{ij}: Rate of signal from i to j

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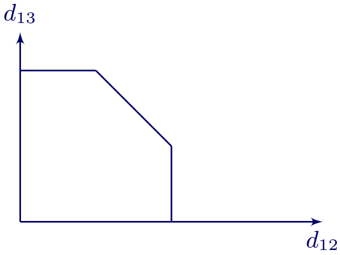

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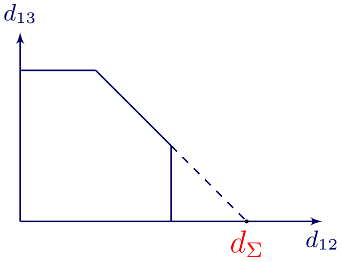

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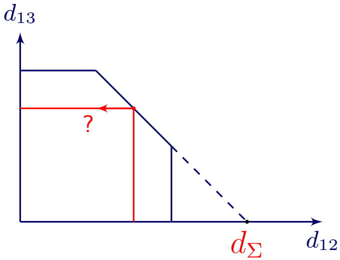

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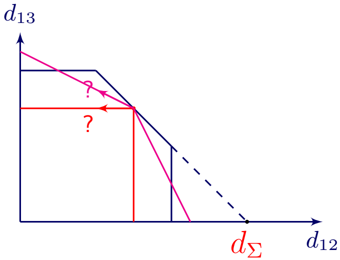

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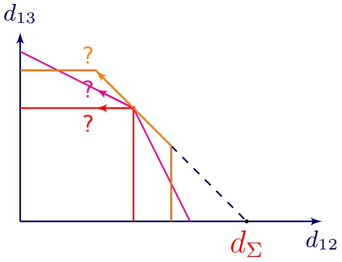

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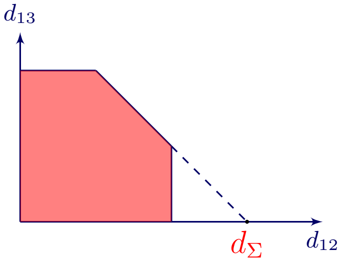

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Goal

Find the DoF region of the MIMO Y-channel.

Main result

The DoF region of a 3-user MIMO Y-channel with $N \leq M$ is described by

$$d_{12} + d_{13} + d_{23} \leq N$$

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The DoF region of a 3-user MIMO Y-channel with $N \leq M$ is described by

$$\begin{aligned} d_{12} + d_{13} + d_{23} &\leq N \\ d_{12} + d_{13} + d_{32} &\leq N \\ \vdots \quad\quad \vdots \quad\quad \vdots \quad &\leq \vdots \end{aligned}$$

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Theorem (DoF region)

DoF region for $N \leq M$ described by

$$d_{p_1 p_2} + d_{p_1 p_3} + d_{p_2 p_3} \leq N, \quad \forall \mathbf{p}$$

where $\mathbf{p}$ is a permutation of $(1, 2, 3)$ and p_i is its i-th component.

Outline

Insight from the upper bounds

Message-flow-graph for upper bounds:

$$d_{p_1 p_2} + d_{p_1 p_3} + d_{p_2 p_3} \leq N$$

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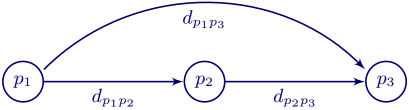

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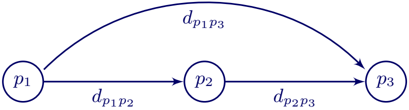
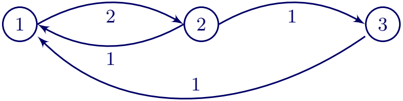


Message-flow-graph for $\boldsymbol{d} = (2, 0, 1, 1, 1, 0) \in \mathcal{D}$:



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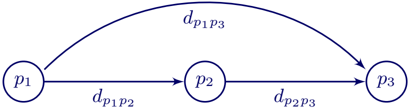

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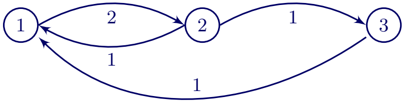

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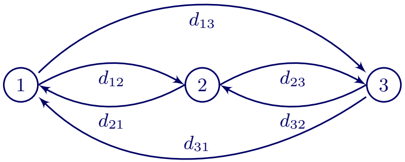

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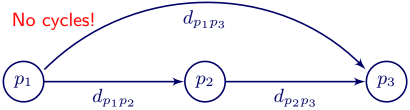

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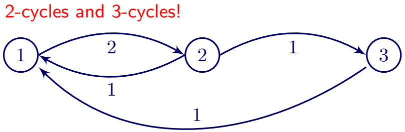

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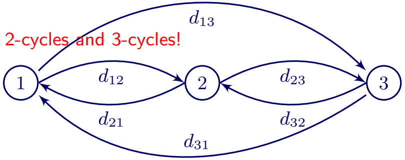

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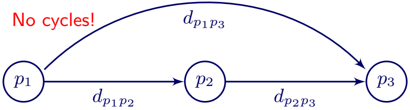

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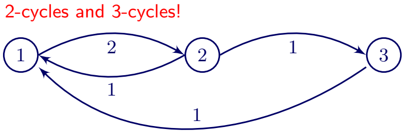

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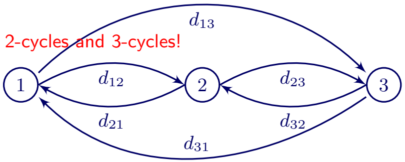
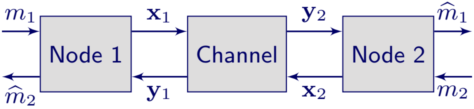


Optimal strategy should be able to 'resolve' cycles!

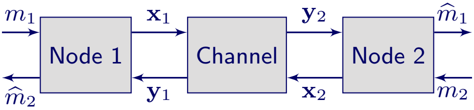
Outline

Uplink-downlink Separability



Two-way channels are in general not separable

Uplink-downlink Separability



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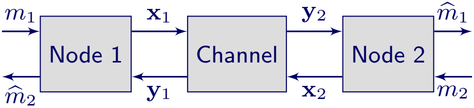

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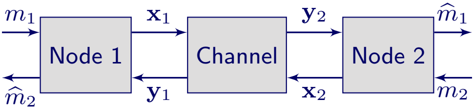

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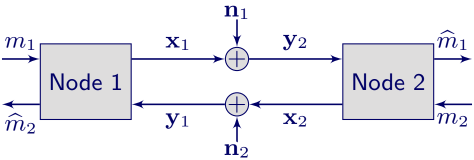
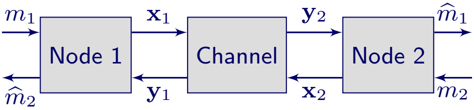


Basic Gaussian two-way channel is separable

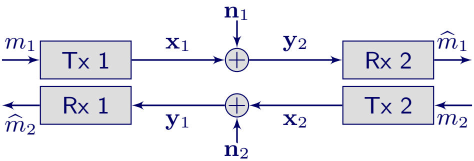
Uplink-downlink Separability



Two-way channels are in general not separable

$\Rightarrow$ uplink and downlink have to be considered jointly

$\Rightarrow$ $\mathbf{x}_i$ and $\mathbf{y}_i$ are dependent



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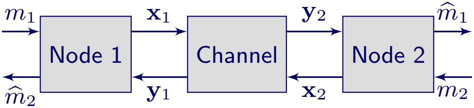

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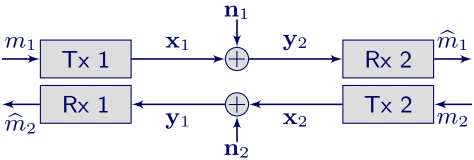

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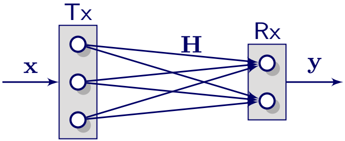

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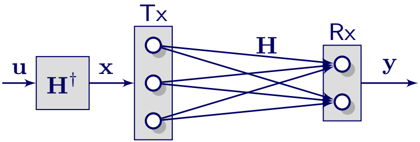

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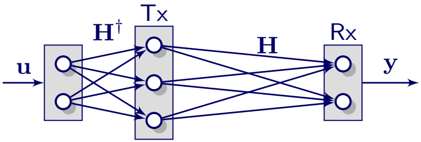

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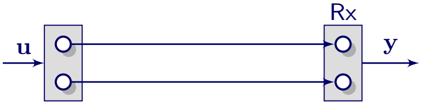

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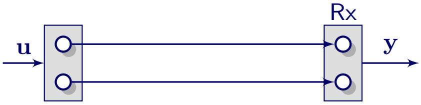

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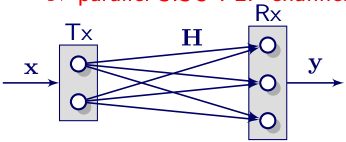

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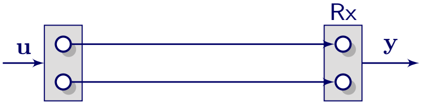

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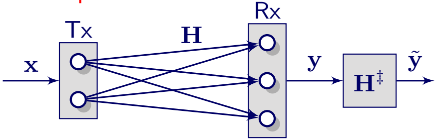

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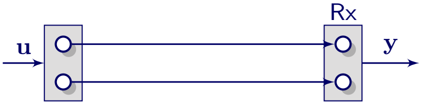

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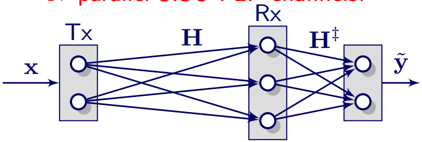

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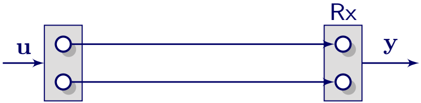

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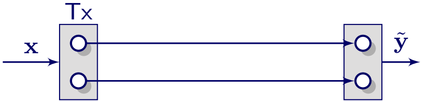

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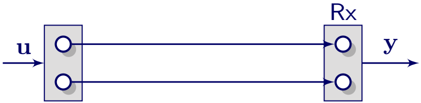

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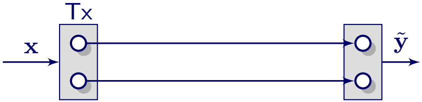

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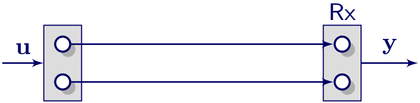

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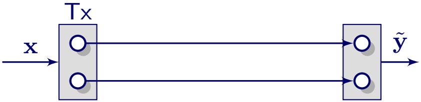

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MAC and BC are also separable

Outline

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Placing two signals $\mathbf{x}_1$ and $\mathbf{x}_2$ in signal space so that $\text{span}(\mathbf{x}_1) = \text{span}(\mathbf{x}_2)$.

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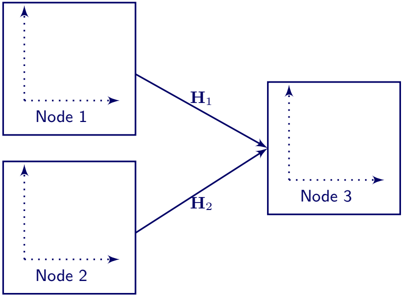

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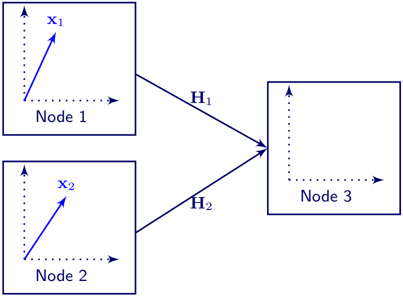

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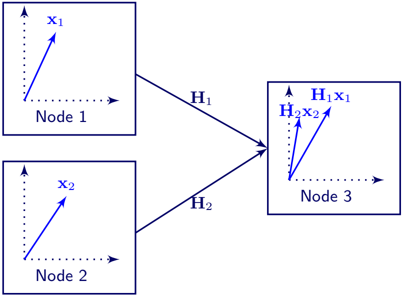

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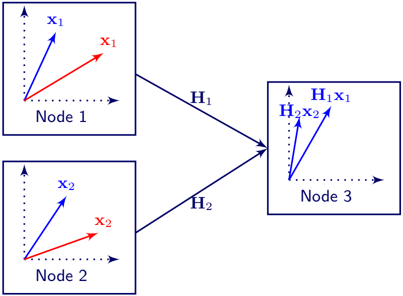

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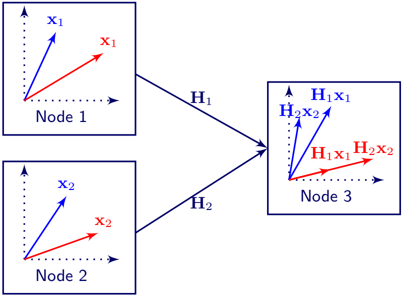

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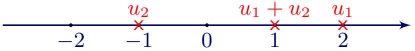

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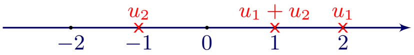

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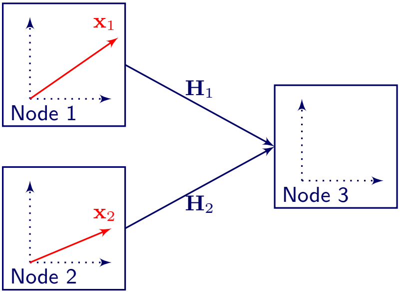

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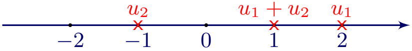

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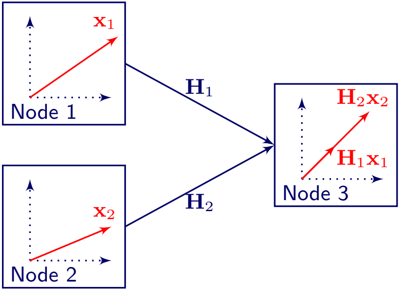

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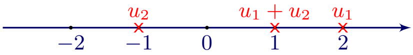
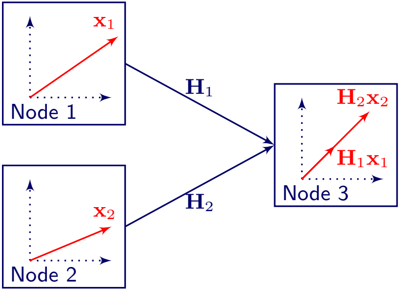
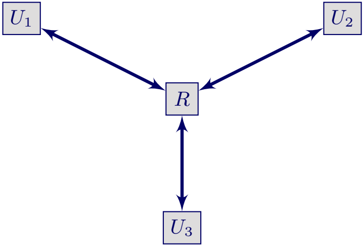




Outline

Overview

Achievability of $\mathcal{D}$ is proved using:



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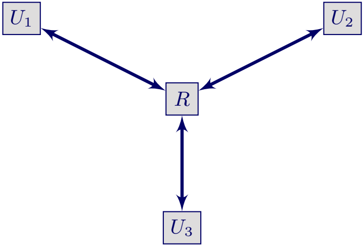
MIMO
Y-channel



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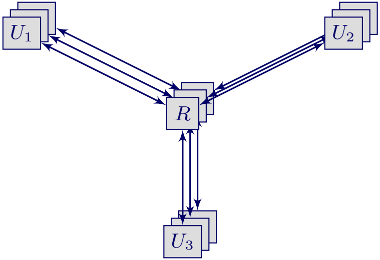
MIMO Y-channel	$\rightarrow$	N SISO Y-channels (sub-channels)



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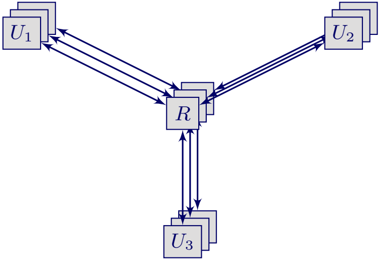

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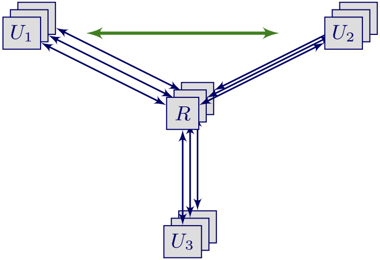

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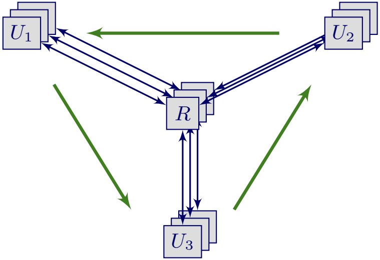

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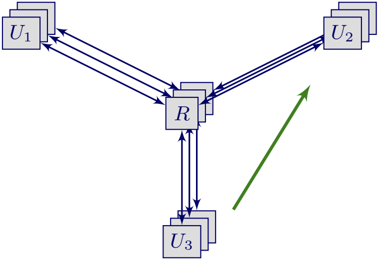

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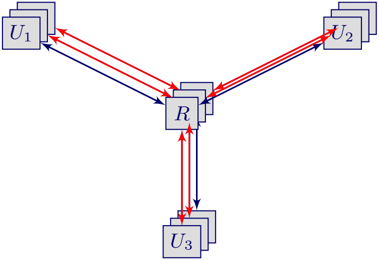

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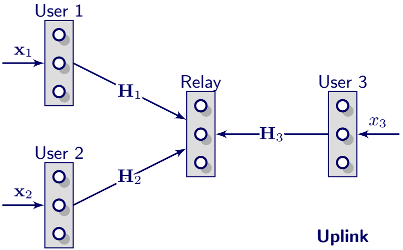
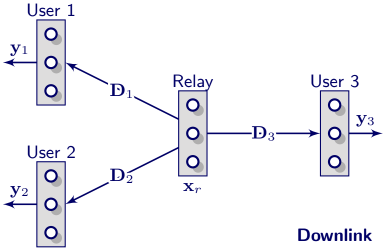
Resource allocation: distribute sub-channels over users

Channel Diagonalization

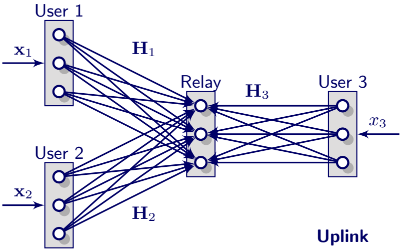
- a MIMO Y-channel with $M = N = 3$

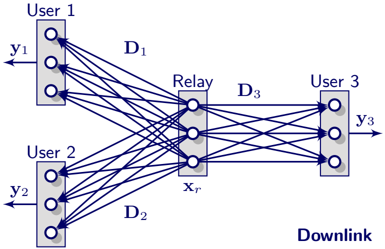




Channel Diagonalization

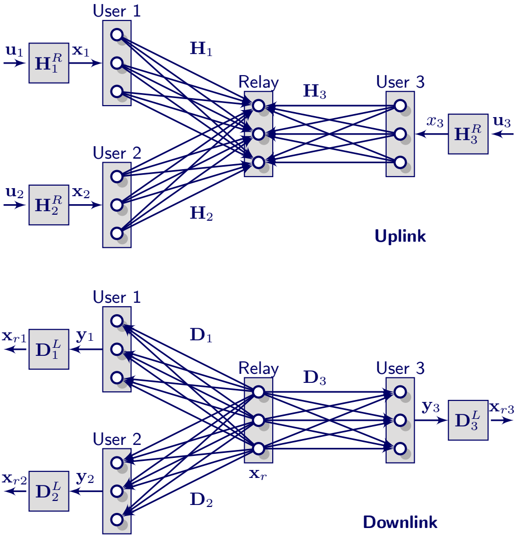
- a MIMO Y-channel with $M = N = 3$
- actually looks like this!





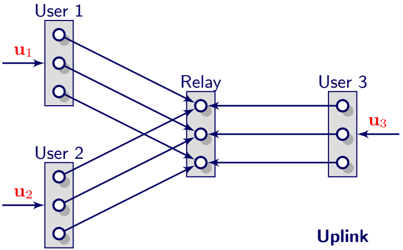
Channel Diagonalization

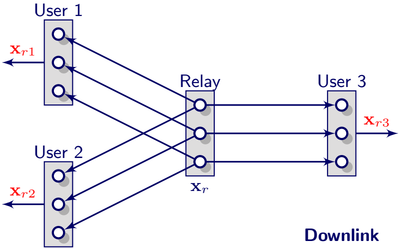
- a MIMO Y-channel with $M = N = 3$
- actually looks like this!
- Pre- and post-code using the Moore-Penrose pseudo inverse



Channel Diagonalization

- a MIMO Y-channel with $M = N = 3$
- actually looks like this!
- Pre- and post-code using the Moore-Penrose pseudo inverse
- Channel Diagonalization $\Rightarrow N$ sub-channels

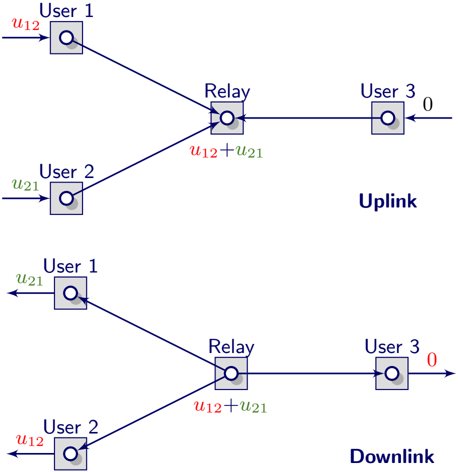




Information transfer

Bi-directional:

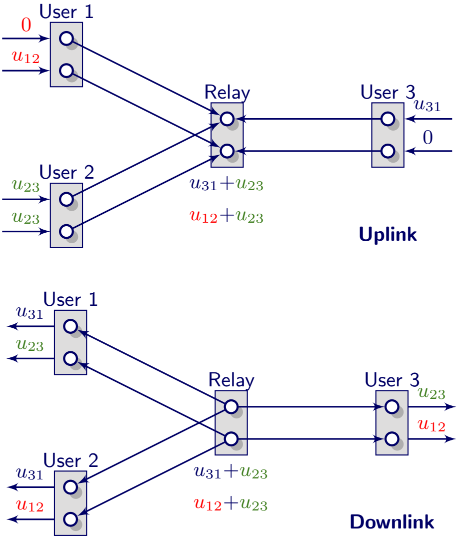
- signal-alignment
- compute-forward
- exchanges 2 symbols
- requires 1 sub-channel (up- and down-link)
- efficiency 2 DoF/dimension



Information transfer

Cyclic:

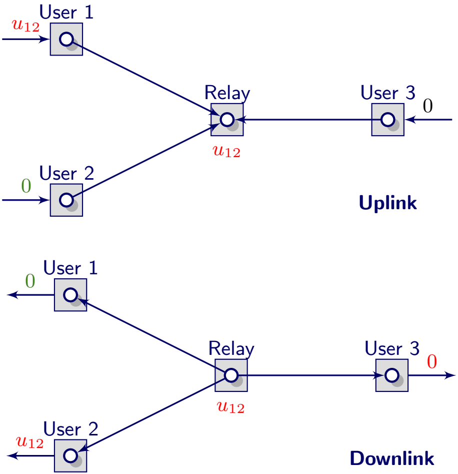
- signal-alignment
- compute-forward
- exchanges 3 symbols
- requires 2 sub-channels (up- and down-link)
- efficiency 3/2 DoF/dimension



Information transfer

Uni-directional:

- decode-forward
- exchanges 1 symbols
- requires 1 sub-channel (up- and down-link)
- efficiency 1 DoF/dimension



Example

DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) = (2, 0, 1, 1, 1, 0)$, Y-channel with $3 = N \leq M$

Example

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Bi-directional + cyclic + uni-directional:

- bi-directional achieves $d_{12}^b = d_{21}^b = 1$ over 1 sub-channel
- residual DoF $(1, 0, 0, 1, 1, 0)$
- cyclic achieves $d_{12}^c = d_{23}^c = d_{31}^c = 1$ over 2 sub-channels

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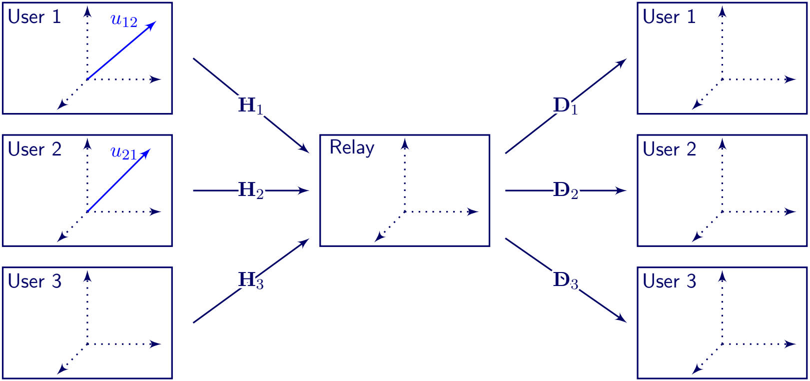
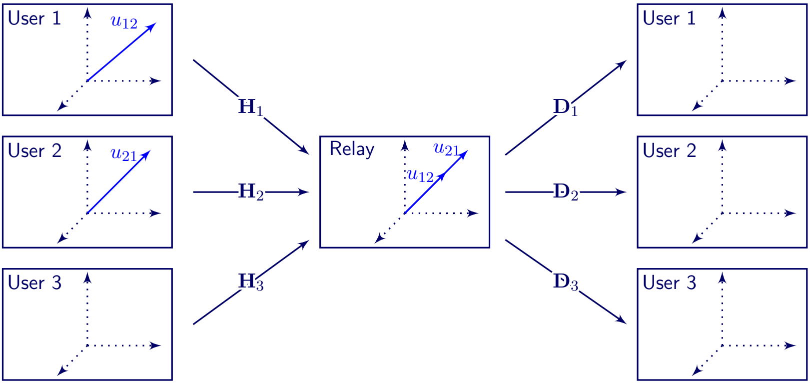
- residual DoF $(1, 0, 0, 1, 1, 0)$
- cyclic achieves $d_{12}^c = d_{23}^c = d_{31}^c = 1$ over 2 sub-channels
- residual DoF $(0, 0, 0, 0, 0, 0)$
- total number of sub-channels 3 $= N$!

Example

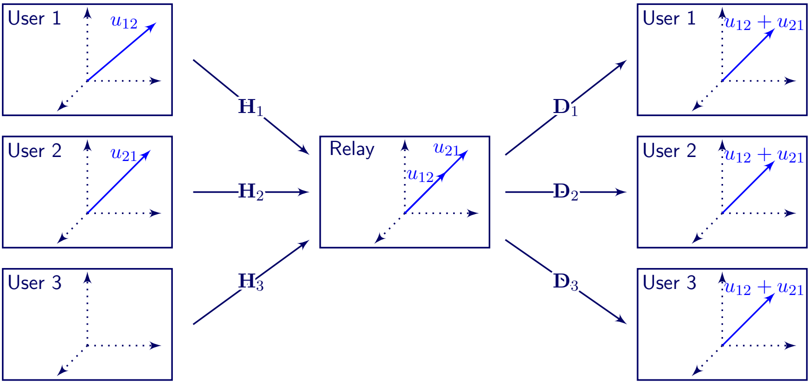

User 1
u_{12}
User 2
u_{21}
User 3
$\mathbf{H}_1$
$\mathbf{H}_2$
$\mathbf{H}_3$
Relay
$\mathbf{D}_1$
$\mathbf{D}_2$
$\mathbf{D}_3$
User 1
User 2
User 3

Example

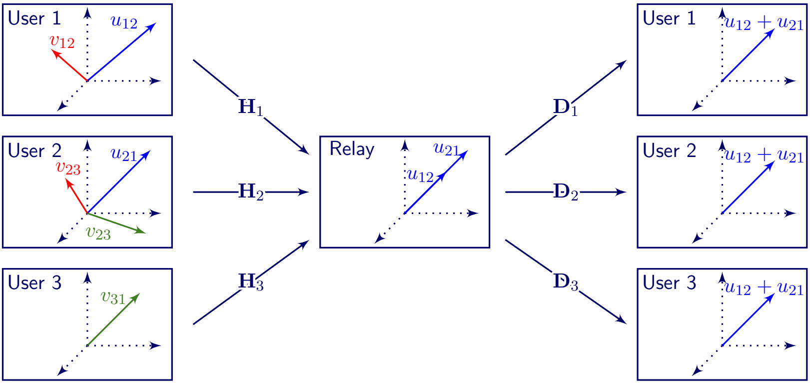


User 1
u_{12}
User 2
u_{21}
User 3
$\mathbf{H}_1$
$\mathbf{H}_2$
$\mathbf{H}_3$
Relay
u_{21}
u_{12}
$\mathbf{D}_1$
$\mathbf{D}_2$
$\mathbf{D}_3$
User 1
User 2
User 3

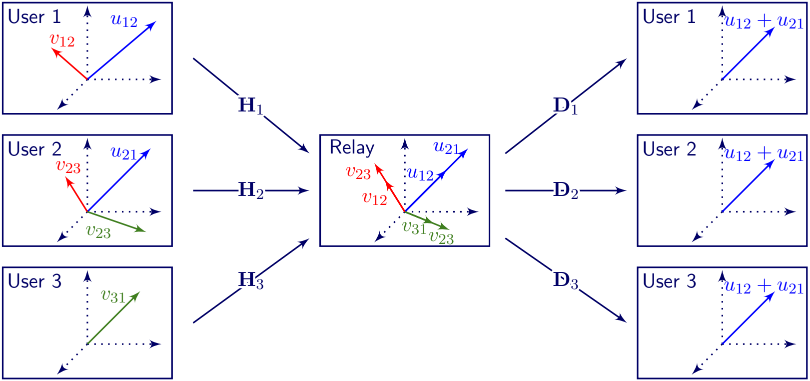
Example


User 1
u_{12}
User 2
u_{21}
User 3
$\mathbf{H}_1$
$\mathbf{H}_2$
$\mathbf{H}_3$
Relay
u_{21}
u_{12}
$\mathbf{D}_1$
$\mathbf{D}_2$
$\mathbf{D}_3$
User 1
$u_{12} + u_{21}$
User 2
$u_{12} + u_{21}$
User 3
$u_{12} + u_{21}$

Example


User 1
v_{12}
u_{12}
User 2
v_{23}
u_{21}
v_{23}
User 3
v_{31}
$\mathbf{H}_1$
$\mathbf{H}_2$
$\mathbf{H}_3$
Relay
u_{21}
u_{12}
$\mathbf{D}_1$
$\mathbf{D}_2$
$\mathbf{D}_3$
User 1
$u_{12} + u_{21}$
User 2
$u_{12} + u_{21}$
User 3
$u_{12} + u_{21}$

Example


User 1
u_{12}
v_{12}
User 2
u_{21}
v_{23}
v_{23}
User 3
v_{31}
$\mathbf{H}_1$
$\mathbf{H}_2$
$\mathbf{H}_3$
Relay
u_{21}
u_{12}
v_{23}
v_{12}
v_{31}
v_{23}
$\mathbf{D}_1$
$\mathbf{D}_2$
$\mathbf{D}_3$
User 1
$u_{12} + u_{21}$
User 2
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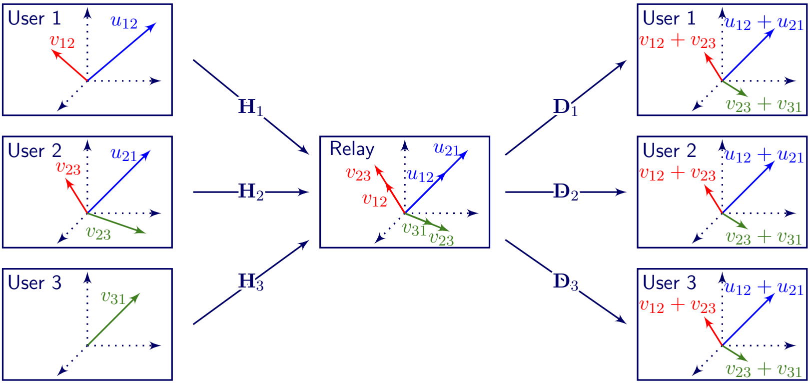
Example


User 1
u_{12}
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u_{21}
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v_{23}
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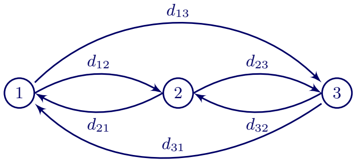
v_{31}
$\mathbf{H}_1$
$\mathbf{H}_2$
$\mathbf{H}_3$
Relay
u_{21}
v_{23}
u_{12}
v_{12}
v_{31}
v_{23}
$\mathbf{D}_1$
$\mathbf{D}_2$
$\mathbf{D}_3$
User 1
$u_{12} + u_{21}$
$v_{12} + v_{23}$
$v_{23} + v_{31}$
User 2
$u_{12} + u_{21}$
$v_{12} + v_{23}$
$v_{23} + v_{31}$
User 3
$u_{12} + u_{21}$
$v_{12} + v_{23}$
$v_{23} + v_{31}$

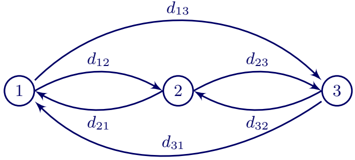
Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$



Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) \Rightarrow$ 2-cycles and 3-cycles!



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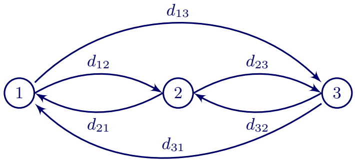
1) set $d_{ij}^b = d_{ji}^b = \min\{d_{ij}, d_{ji}\}$



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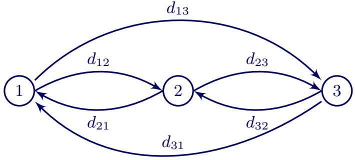

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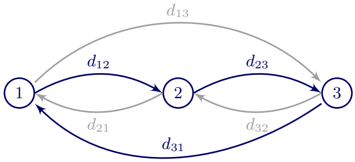

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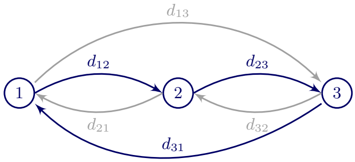
1) set $d^b_{ij} = d^b_{ji} = \min\{d_{ij}, d_{ji}\}$
2) requires d^b_{ij} sub-channels
3) resolves 2-cycles
4) residual DoF $d'_{ij} = d_{ij} - d^b_{ij}$



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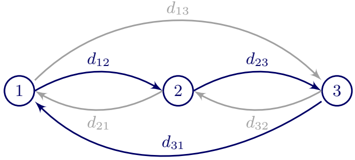

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Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0)$ ⇒ 3-cycle!

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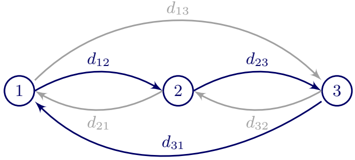

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Resource allocation

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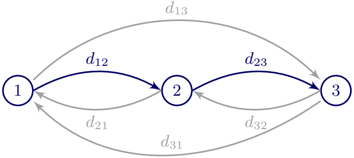

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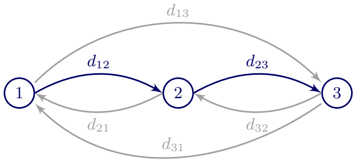
Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0)$ ⇒ 3-cycle!

Cyclic:

1) set $d^c_{ij} = d^c_{jk} = d^c_{ki} = \min\{d'_{ij}, d'_{jk}, d'_{ki}\}$
2) requires $2d^c_{ij}$ sub-channels

Resource allocation

Consider a DoF tuple $\mathbf{d} = (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32})$ ⇒ 2-cycles and 3-cycles!



Bi-directional:

1) set $d_{ij}^b = d_{ji}^b = \min\{d_{ij}, d_{ji}\}$
2) requires d_{ij}^b sub-channels
3) resolves 2-cycles
4) residual DoF $d'_{ij} = d_{ij} - d_{ij}^b$

Residual DoF tuple (e.g.) $\mathbf{d}' = (d'_{12}, 0, 0, d'_{23}, d'_{31}, 0)$ ⇒ 3-cycle!

Cyclic:

1) set $d_{ij}^c = d_{jk}^c = d_{ki}^c = \min\{d'_{ij}, d'_{jk}, d'_{ki}\}$
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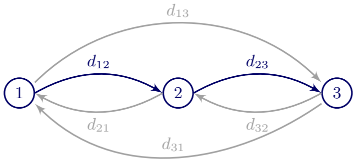

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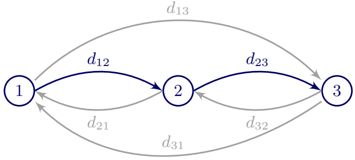

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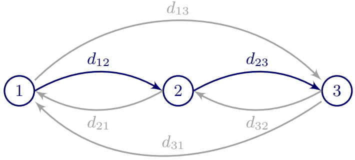

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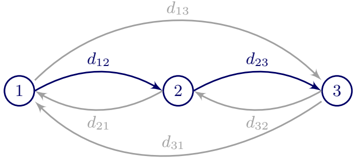

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$\mathbf{d}$ achieved!

Outline

1. Motivation: From one-way to multi-way
2. The MIMO Y-channel: From single-antenna to multiple-antennas
3. Main result: From capacity to DoF
4. Insights and ingredients
    - Channel diagonalization: Separability of communication structure
    - Alignment, Compute-and-forward
    - Transmission strategy(3-users)
5. **Extensions and Conclusion**

K-user Case

For the K-user Y-channel with $N \leq M$:

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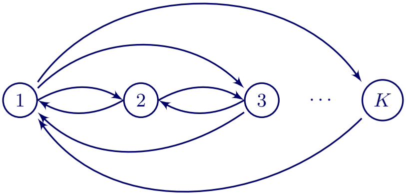

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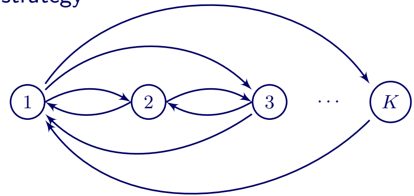

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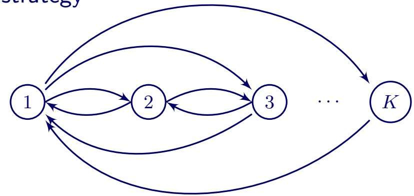

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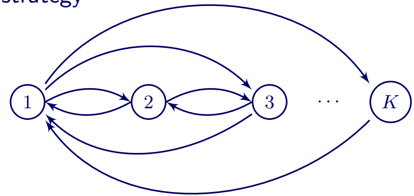

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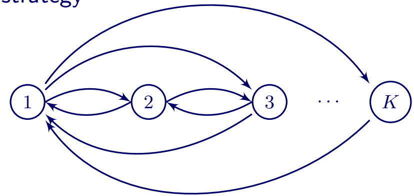

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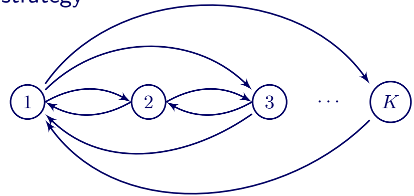

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- DoF region described by

$$\sum_{i=1}^{K-1} \sum_{j=i+1}^{K} d_{p_i p_j} \leq N, \quad \forall \mathbf{p}$$

where $\mathbf{p}$ is a permutation of $(1, 2, \cdots, K)$.

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Thank You

Related work

1. C. Shannon, *Two-way communication channels*, Proc. of Fourth Berkeley Symposium on Mathematics, Statistics, and Probability, 1961.
2. B. Rankov and A. Wittneben, *Spectral efficient signaling for half-duplex relay channels*, Proc. of the Asilomar Conference on Signals, Systems, and Computers, 2005.
3. D. Gündüz, A. Yener, A. Goldsmith, and H. V. Poor, *The multi-way relay channel*, IEEE Transactions on Information Theory, Vol. 59(1), pp. 51-63.
4. N. Lee, J.-B. Lim, and J. Chun, *Degrees of freedom of the MIMO Y channel: Signal space alignment for network coding*, IEEE Trans. on Info. Theory, 2010.
5. A. Chaaban and A. Sezgin,, *Approximate Sum-Capacity of the Y-channel*, IEEE Transactions on Information Theory, Vol.59(9), pp.5723-5740.
6. A. Chaaban and A. Sezgin,, *Multi-way communications*, Foundations and Trends in Communications and Information Theory, now publishers, in review.

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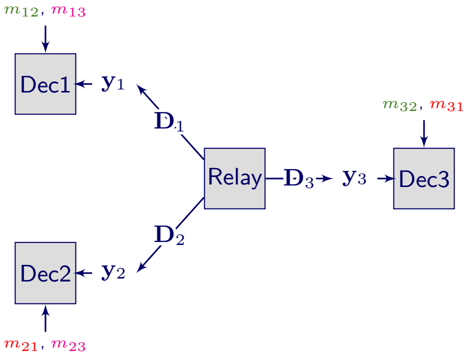
$$= d_{12} + d_{23} + d_{31} - d_{12}^c \qquad (d_{12}^c = d_{12} - d_{12}^b \text{ e.g.})$$

$$= d_{12}^b + d_{23} + d_{31}$$

$$= d_{21} + d_{23} + d_{31}$$

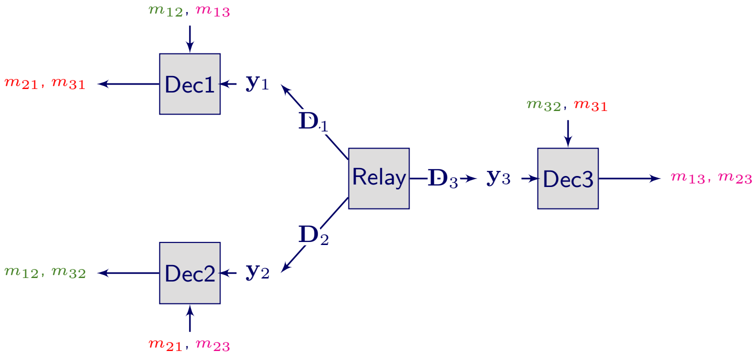
No cycles $\Rightarrow N_s \leq N \Rightarrow$ All $\mathbf{d} \in \mathcal{D}$ are achievable

Outer bound



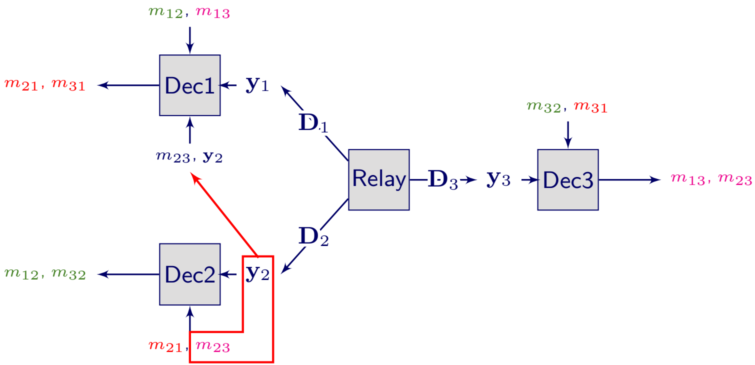
Consider any reliable scheme for the 4-user MIMO MRC

Outer bound



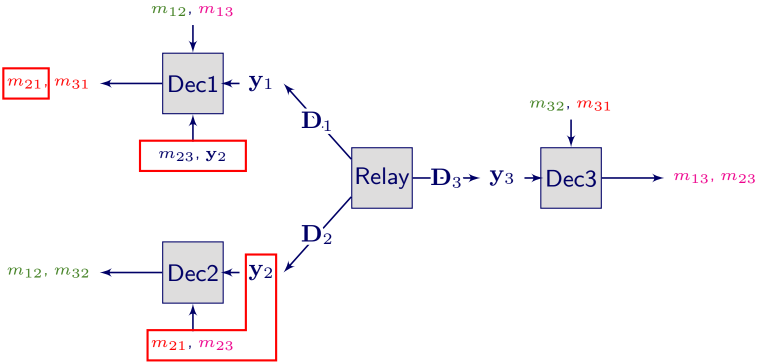
Users can decode their desired signals

Outer bound



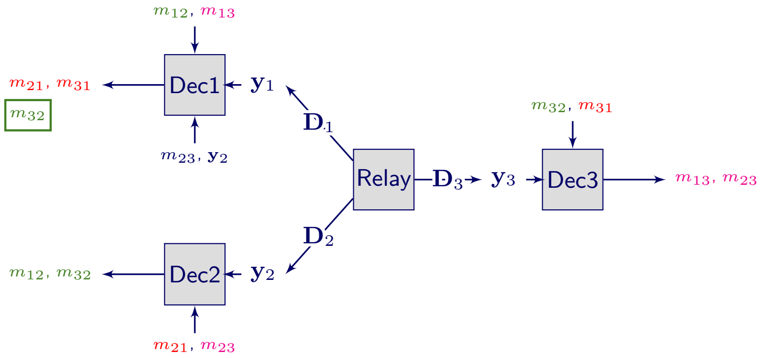
Give m_{23} and $\mathbf{y}_2$ to user 1 as side info.

Outer bound



Now, user 1 has the info. available at user 2

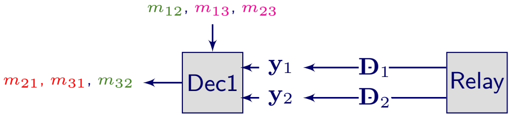
Outer bound



$\Rightarrow$ User 1 can decode m_{32}

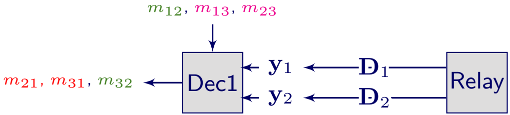
Upper bound

User 1 can decode (m_{21}, m_{31}, m_{32}) from $(m_{12}, m_{13}, \mathbf{y}_1, \overbrace{m_{23}, \mathbf{y}_2}^{\text{side info.}})$



Upper bound

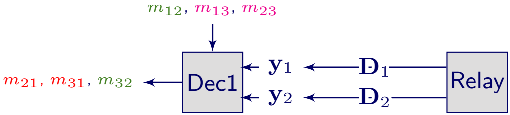
User 1 can decode (m_{21}, m_{31}, m_{32}) from $(m_{12}, m_{13}, \mathbf{y}_1, \overbrace{m_{23}, \mathbf{y}_2}^{\text{side info.}})$



$$\Rightarrow R_{21} + R_{31} + R_{32} \leq I(\mathbf{x}_r; \mathbf{y}_1, \mathbf{y}_2) = I\left(\mathbf{x}_r; \begin{bmatrix}\mathbf{D}_1\\ \mathbf{D}_2\end{bmatrix}\mathbf{x}_r + \begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix}\right)$$ P2P Channel

Upper bound

User 1 can decode (m_{21}, m_{31}, m_{32}) from $(m_{12}, m_{13}, \mathbf{y}_1, \overbrace{m_{23}, \mathbf{y}_2}^{\text{side info.}})$



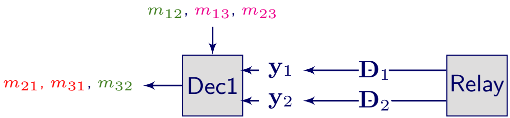
$$\Rightarrow R_{21} + R_{31} + R_{32} \leq I(\mathbf{x}_r; \mathbf{y}_1, \mathbf{y}_2) = I\left(\mathbf{x}_r; \begin{bmatrix}\mathbf{D}_1\\ \mathbf{D}_2\end{bmatrix} \mathbf{x}_r + \begin{bmatrix}\mathbf{z}_1\\ \mathbf{z}_2\end{bmatrix}\right)$$

P2P Channel

$$\Rightarrow d_{21} + d_{31} + d_{32} \leq \text{rank}\left(\begin{bmatrix}\mathbf{D}_1\\ \mathbf{D}_2\end{bmatrix}\right) = N$$

Upper bound

User 1 can decode (m_{21}, m_{31}, m_{32}) from $(m_{12}, m_{13}, \mathbf{y}_1, \overbrace{m_{23}, \mathbf{y}_2}^{\text{side info.}})$



$$\Rightarrow R_{21} + R_{31} + R_{32} \leq I(\mathbf{x}_r; \mathbf{y}_1, \mathbf{y}_2) = I\left(\mathbf{x}_r; \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix} \mathbf{x}_r + \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}\right)$$ P2P Channel

$$\Rightarrow d_{21} + d_{31} + d_{32} \leq \text{rank}\left(\begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix}\right) = N$$

Considering different combinations of users gives the desired outer bound

$$\sum_{i=1}^{2} \sum_{j=i+1}^{3} d_{p_i p_j} \leq N, \quad \forall \mathbf{p}$$