How to Handle Antenna Coupling in MIMO Systems

B. Hochwald, D. Nie and E. Stauffer

University of Notre Dame

bhochwald@nd.edu

August 1, 2014

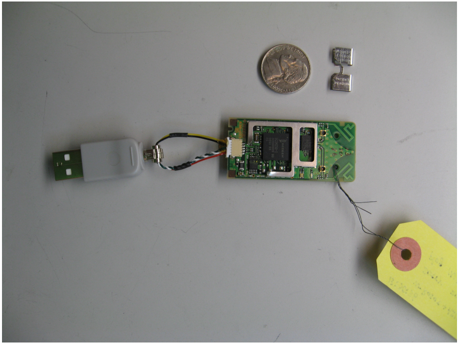
Overview

1. Background
   - Introduction to Coupling and Matching
   - Decoupling Multiple Antennas
2. Systematic Design of Decoupling Networks
   - Properties of Decoupling Networks
   - Design Methods for Simple Decoupling Networks
   - Other Design Methods
3. Where Work is Needed
   - High-Bandwidth Decoupling Network Design
   - Theoretical Bounds on Bandwidth
   - Lossy Matching Networks
4. Summary

Overview

1. Background
   - Introduction to Coupling and Matching
   - Decoupling Multiple Antennas
2. Systematic Design of Decoupling Networks
   - Properties of Decoupling Networks
   - Design Methods for Simple Decoupling Networks
   - Other Design Methods
3. Where Work is Needed
   - High-Bandwidth Decoupling Network Design
   - Theoretical Bounds on Bandwidth
   - Lossy Matching Networks
4. Summary

Radio-Frequency Coupling Happens



$$S_{12} = S_{21} \neq 0$$

Becoming Increasingly Important

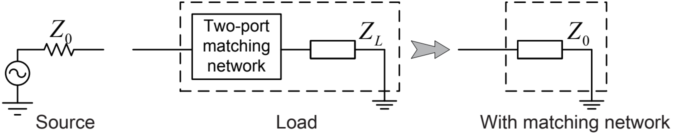
Where it can happen:

- Compact devices
- Wearables
- Massive MIMO
- At all frequencies, WiFi, LTE, 5G technologies

Take-aways from this talk:

- Matching circuits can compensate
- Envelope correlation can still be made zero
- System bandwidth and capacity can still be high
- Don't over-engineer antennas to eliminate coupling

Matching Network



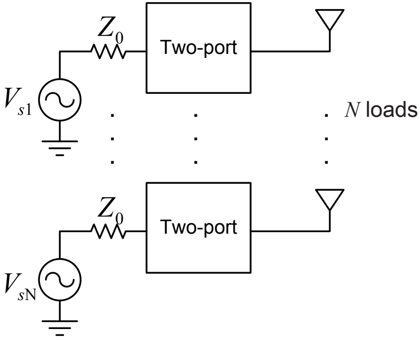
Without matching network, reflection coefficient is

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

With matching network, reflection coefficient is zero.

Matching Multiple Loads

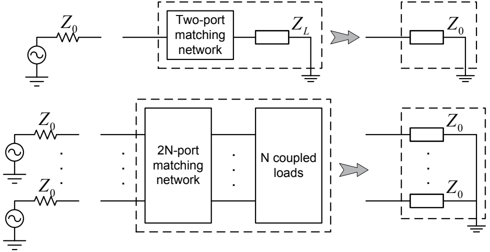
Multiple two-port networks?



This only works if the loads are “uncoupled”. Power from one chain is still coupled into another.

Decoupling Network

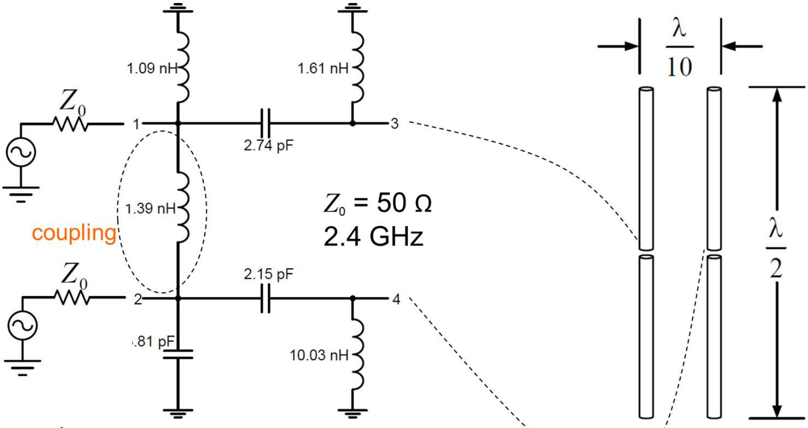
- Matches uncoupled sources to coupled loads
- Transforms the coupled impedance of the loads into the uncoupled characteristic impedance of the sources
- Ensures no signal entering one port is reflected out another



Needed:

- Theory for decoupling network design
- Theory for decoupling network bandwidth analysis
- Focus on:
  - Simplicity
  - High bandwidth
  - Implementation with lossy components

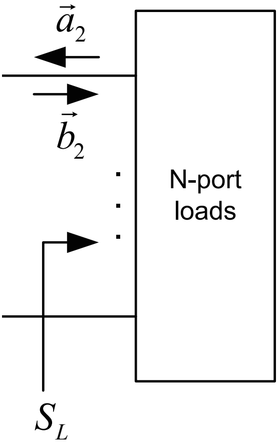
Example for Two Loads



Observations:

- Coupled matching network
- Asymmetric
- Minimum complexity

Arbitrary Coupled Loads

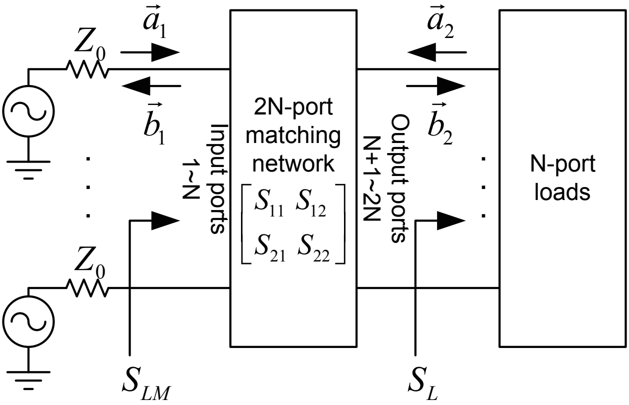


Specified by a scattering matrix (S-matrix)

$$\vec{a}_2 = S_L \vec{b}_2$$

Non-zero off-diagonal entries indicate coupling.

Definition of Decoupling Network

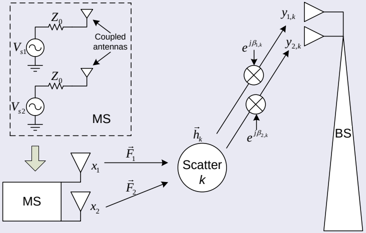


- Designed for a given scattering matrix of loads S_L
- Permits no reflected energy from loads
- Combined scattering matrix of network+loads $S_{LM} = 0$

Effect of Network In a MIMO System

Channel Model

- N transmit antennas and M receive antennas
- Mutual coupling in N transmit antennas
- Uplink from mobile station to base station is in rich scatter environment



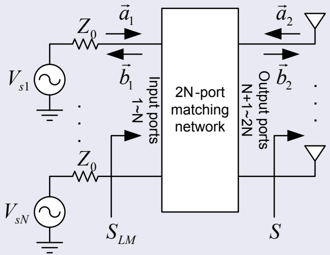
- $M \times N$ channel
  $\vec{y} = H\vec{x} + \vec{w}$
- Channel matrix
  $H = \tilde{H}R^{1/2}$
- $\tilde{H}$: an $M \times N$ matrix whose elements are i.i.d. $\mathcal{CN}(0,1)$

$$R = \frac{1}{2\eta}\int_{4\pi} \times \begin{bmatrix} |\vec{F}_1|^2 & \vec{F}_1^* \cdot \vec{F}_2 & \cdots & \vec{F}_1^* \cdot \vec{F}_N \\ \vec{F}_2^* \cdot \vec{F}_1 & |\vec{F}_2|^2 & \cdots & \vec{F}_2^* \cdot \vec{F}_N \\ \vdots & \vdots & \ddots & \vdots \\ \vec{F}_N^* \cdot \vec{F}_1 & \vec{F}_N^* \cdot \vec{F}_2 & \cdots & |\vec{F}_N|^2 \end{bmatrix} d\Omega$$

Capacity in MIMO System

The Channel Correlation Matrix

- $0 \leq R \leq I$ in positive-definite sense
- R is decided by the matching network
- A decoupling network decorrelates the channel by making $R = I$



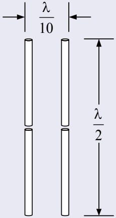
Channel Capacity

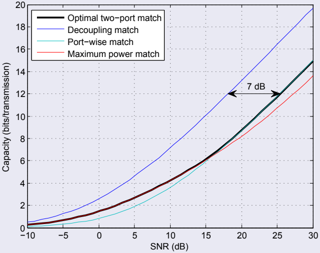
Capacity of the channel is

$$C_p = \mathbb{E}[\log\det(I + \frac{1}{\sigma^2} HH^H)] = \mathbb{E}[\log\det(I + \frac{1}{\sigma^2} \tilde{H} R \tilde{H}^H)]$$

Capacity as a Function of Matching Network

Decoupling Networks





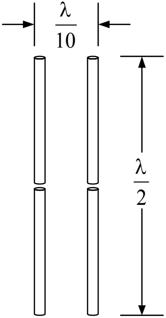
- When the decoupling network is used, the capacity is maximized
- There is a 7 dB gap between the decoupling network and two-port matching networks

- Effectively eliminates the mutual coupling as seen by source
- Decorrelates the channel
- Maximizes the radiated power and capacity

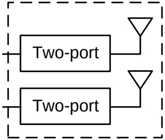
Connection Between Envelope Correlation and Antenna Coupling

Envelope correlation

- Defined as the cross-correlation of the antenna patterns integrated over all space
- Measure of signal correlation seen at baseband
- Measure of antenna diversity



- Closely spaced antenna pairs
- High coupling



- What is effect of matching network on envelope correlation?

Envelope Correlation of Antennas

Envelope Correlation

Defined as

$$\rho = \frac{\left| \int_{4\pi} \vec{F}_1 \cdot \vec{F}_2^* \, d\Omega \right|^2}{\int_{4\pi} |\vec{F}_1|^2 d\Omega \int_{4\pi} |\vec{F}_2|^2 d\Omega}$$

- The envelope correlation is related to the S-parameters as

$$\rho = \frac{|S_{11}^* S_{12} + S_{21}^* S_{22}|^2}{(1 - |S_{11}|^2 - |S_{21}|^2)(1 - |S_{12}|^2 - |S_{22}|^2)}$$

- If the antennas are matched, $S_{11} = S_{22} = 0$, then the envelope correlation is zero
- If the antennas are decoupled, $S_{12} = S_{21} = 0$, then the envelope correlation is also zero

Mathematical Description of Decoupling Network

- Partitioned $2N \times 2N$ S-matrix:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

- Reciprocal: $S = S^T$
- Lossless: $S^H S = I$
- Matches loads at output: $S_{22} = S_L^H$
- Decouples: $S_{11} + S_{12} S_L (I - S_L^H S_L)^{-1} S_{21} = 0$

Constraints leave N^2 degrees of freedom in S, so S is not unique.
Open question: Which is the best one?

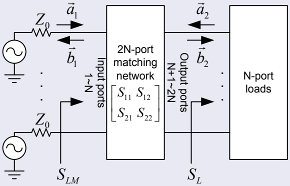
Overview

Properties of Decoupling Networks

Decoupling Network

Decoupling network is lossless, reciprocal, $2N$-port network S that satisfies $S_{LM} = 0$, where

$$S_{LM} = S_{11} + S_{12} S_L (I - S_{22} S_L)^{-1} S_{21}$$



Non-uniqueness of Decoupling Networks

- Set of decoupling networks for S_L

$$\mathcal{S} := \{S \in \mathbb{C}^{2N \times 2N} : S_{22} = S_L^H, S^H S = I, S^T = S\}$$

- $\mathcal{S}$ has N^2 degrees of freedom
- Let SVD $S_L = U_L \Lambda_L V_L^H$. Unitary matrix V is used to represents N^2 DoF

$$S_{11} = -V V_L^H U_L^* \Lambda_L V^T$$

$$S_{12} = V(I - \Lambda_L^2)^{\frac{1}{2}} U_L^H$$

$$S_{21} = U_L^*(I - \Lambda_L^2)^{\frac{1}{2}} V^T$$

$$S_{22} = V_L \Lambda_L U_L^H$$

Properties of Decoupling Networks

- The admittance matrix and S-matrix are related by Cayley transform

$$Y = Y_0(I - S)(I + S)^{-1}$$

Lemma

With $S_L = U_L \Lambda_L V_L^H$, then $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ represents the admittance of a decoupling network for S_L if and only if it has the form

$$Y_{11} = Y_0(V^* A^T + V A^H)(V^* A^T - V A^H)^{-1}$$

$$Y_{12} = Y_{21}^T = -2Y_0(AV^H - A^* V^T)^{-1}$$

$$Y_{22} = Y_0(BV^H + B^* V^T)(AV^H - A^* V^T)^{-1}$$

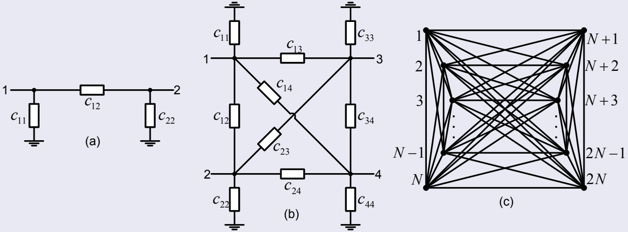
for some unitary matrix V, where

$$A = U_L(I - \Lambda_L^2)^{-\frac{1}{2}} + V_L(I - \Lambda_L^2)^{-\frac{1}{2}} \Lambda_L$$

$$B = U_L(I - \Lambda_L^2)^{-\frac{1}{2}} - V_L(I - \Lambda_L^2)^{-\frac{1}{2}} \Lambda_L$$

Network Synthesis From Admittance Matrix

Generalized Π-Network



- Generalized $2N$-port Π-network:

$$Y = \begin{bmatrix} \sum_{i=1}^{2N} c_{1i} & -c_{12} & \cdots & -c_{1(2N)} \\ -c_{12} & \sum_{i=1}^{2N} c_{2i} & \cdots & -c_{2(2N)} \\ \vdots & \vdots & \ddots & \vdots \\ -c_{1(2N)} & -c_{2(2N)} & \cdots & \sum_{i=1}^{2N} c_{i(2N)} \end{bmatrix}$$

- Π-network:

$$Y = \begin{bmatrix} c_{11} + c_{12} & -c_{12} \\ -c_{12} & c_{12} + c_{22} \end{bmatrix}$$

Network Complexity

- Let $\mathbf{I}(Y)$ be the number of nonzero components in the realization of Y
- For general dense Y, $\mathbf{I}(Y) = 2N^2 + N$
- We design simplest decoupling network by solving

$$Y^\star = \underset{Y:(Y_0 I - Y)(Y_0 I + Y)^{-1} \in \mathcal{S}}{\arg\min} \mathbf{I}(Y)$$

- $\mathcal{S}$ has N^2 degrees of freedom, so there is a lower bound $\mathbf{I}(Y^\star) \geq \mathbf{I}^\star$

$$\mathbf{I}^\star = 2N^2 + N - N^2 = N^2 + N.$$

- A design method is introduced to achieve this lower bound

Design Method for Simple Decoupling Networks

Minimum-complexity method

Follow the steps below:

1. Calculate $N \times N$ complex matrices P and Q using:
$$P = (I - S_L S_L^H)^{-1}, \quad Q = S_L^H (I - S_L S_L^H)^{-1}$$
and use p_{ij}, q_{ij} to denote the ijth element of P and Q.
2. If $N \geq 3$, solve the following quadratic equation for real θ_1:
$$d_1 \tan^2 \theta_1 + d_2 \tan \theta_1 + d_3 = 0$$
where
$$\begin{aligned} d_1 &= \gamma_{21}(\delta_{N1}\gamma_{N2} - \gamma_{N1}\delta_{N2}) - \delta_{21}(\delta_{N1}\alpha_{N2} - \gamma_{N1}\beta_{N2}) \\ d_2 &= \alpha_{21}(\delta_{N1}\gamma_{N2} - \gamma_{N1}\delta_{N2}) - \delta_{21}(\beta_{N1}\alpha_{N2} - \alpha_{N1}\beta_{N2}) \\ &\quad + \gamma_{21}(\beta_{N1}\gamma_{N2} - \alpha_{N1}\delta_{N2}) - \beta_{21}(\delta_{N1}\alpha_{N2} - \gamma_{N1}\beta_{N2}) \\ d_3 &= \alpha_{21}(\beta_{N1}\gamma_{N2} - \alpha_{N1}\delta_{N2}) - \beta_{21}(\beta_{N1}\alpha_{N2} - \alpha_{N1}\beta_{N2}) \end{aligned}$$
$$\alpha_{ij} = \mathrm{Re}\{p_{ij} + q_{ij}\}, \quad \beta_{ij} = \mathrm{Im}\{-p_{ij} + q_{ij}\},$$
$$\gamma_{ij} = \mathrm{Im}\{p_{ij} + q_{ij}\}, \quad \delta_{ij} = \mathrm{Re}\{p_{ij} - q_{ij}\}.$$
If a real solution for θ_1 does not exist or $N = 2$, set $\theta_1 = \pi/2$.

Minimum Complexity Method

Method (cont'd)

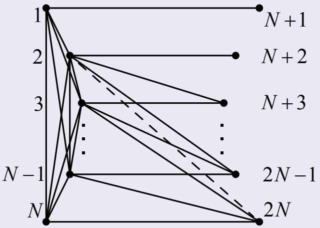
3. Calculate real $\theta_2, \ldots, \theta_N$ using
$$\tan\theta_i = \frac{|p_{i1}|\cos(\angle p_{i1} - \theta_1) + |q_{i1}|\cos(\angle q_{i1} - \theta_1)}{|p_{i1}|\sin(\angle p_{i1} - \theta_1) - |q_{i1}|\sin(\angle q_{i1} - \theta_1)}$$
and let $\Theta = \mathrm{diag}(\theta_1, \cdots, \theta_N)$ be an $N \times N$ diagonal matrix.
4. Use the Cholesky factorization to find an $N \times N$ real lower triangular matrix L_c that is non-negative along the diagonal and satisfies
$$L_c L_c^T = 2\mathrm{Re}\{e^{j\Theta} P e^{-j\Theta}\} + 2\mathrm{Re}\{e^{-j\Theta} Q e^{-j\Theta}\} - I$$
5. Let $Y^\star = \begin{bmatrix} Y_{11}^\star & Y_{12}^\star \\ Y_{21}^\star & Y_{22}^\star \end{bmatrix}$ where
$$Y_{11}^\star = jY_0 L_c^{-1}\mathrm{Re}\{2\mathrm{Re}\{e^{j\Theta}P\} + 2\mathrm{Re}\{e^{-j\Theta}Q\} - e^{-j\Theta}\}(\csc\Theta)L_c$$
$$Y_{12}^\star = (Y_{21}^\star)^T = -jY_0 L_c^T \csc\Theta$$
$$Y_{22}^\star = jY_0 \cot\Theta$$
6. Realize the Π-network circuit using $Y^\star$

Theorem for Minimum Complexity Method

Theorem

$Y^\star =$

$$
\left[\begin{array}{cccccc|cccccc}
\times & \times & \times & \cdots & \times & \times & \times & 0 & 0 & \cdots & 0 & 0 \\
\times & \times & \times & \cdots & \times & \times & 0 & \times & \times & \cdots & \times & \star \\
\times & \times & \times & \cdots & \times & \times & 0 & 0 & \times & \cdots & \times & \times \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\times & \times & \times & \cdots & \times & \times & 0 & 0 & 0 & \cdots & \times & \times \\
\times & \times & \times & \cdots & \times & \times & 0 & 0 & 0 & \cdots & 0 & \times \\
\hline
\times & 0 & 0 & \cdots & 0 & 0 & \times & 0 & 0 & \cdots & 0 & 0 \\
0 & \times & 0 & \cdots & 0 & 0 & 0 & \times & 0 & \cdots & 0 & 0 \\
0 & \times & \times & \cdots & 0 & 0 & 0 & 0 & \times & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \times & \times & \cdots & \times & 0 & 0 & 0 & 0 & \cdots & \times & 0 \\
0 & \star & \times & \cdots & \times & \times & 0 & 0 & 0 & \cdots & 0 & \times
\end{array}\right]
$$



Symmetric Simplification Method

- Symmetric loads have the form

$$S_L = \begin{bmatrix} \mu_L + \xi_L & \xi_L & \cdots & \xi_L \\ \xi_L & \mu_L + \xi_L & \cdots & \xi_L \\ \vdots & \vdots & \ddots & \vdots \\ \xi_L & \xi_L & \cdots & \mu_L + \xi_L \end{bmatrix}$$

- A realization of decoupling network with linear number of components?

Symmetric Simplification Method

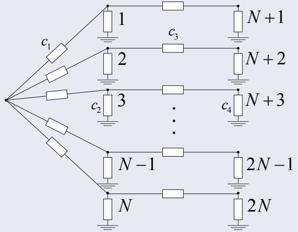
Follow the steps below:

1. Calculate $\mu_1, \xi_1, \mu_2, \xi_2$ as follows:

$$\mu_1 = \frac{1}{1-|\mu_L|^2}$$
$$\xi_1 = -\frac{\mu_L \xi_L^* + \mu_L^* \xi_L + N|\xi_L|^2}{(1-|\mu_L|^2)(N\mu_L\xi_L^* + N\mu_L^*\xi_L + |\mu_L|^2 + N^2|\xi_L|^2 - 1)}$$
$$\mu_2 = \mu_L^* \mu_1$$
$$\xi_2 = \mu_L^* \xi_1 + \xi_L^* \mu_1 + N\xi_L^* \xi_1$$

2. Compute real $\theta_1 = \dfrac{\angle\xi_2 - \arccos(-\frac{\mathrm{Re}\{\xi_1\}}{|\xi_2|})}{2}$
3. Let

$$\mu_{11} = \frac{2\mathrm{Re}\{e^{-j\theta_1}\mu_1 + e^{-j\theta_1}\mu_2\} - \cos\theta_1}{\sin\theta_1}$$
$$\xi_{11} = \frac{2\mathrm{Re}\{e^{-j\theta_1}\xi_1 + e^{-j\theta_1}\xi_2\}}{\sin\theta_1}$$
$$\mu_{12} = \mu_{21} = -\frac{\sqrt{2\mathrm{Re}\{\mu_1 + \xi_1 + e^{-j2\theta_1}(\mu_2 + \xi_2)\} - 1}}{\sin\theta_1}$$
$$\mu_{22} = \cot\theta_1$$



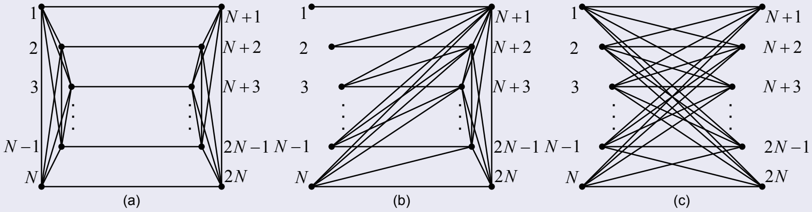
Then realize the network using the components

$$c_1 = -jY_0 N\xi_{11}$$
$$c_2 = jY_0(\mu_{11} + N\xi_{11} + \mu_{12})$$
$$c_3 = -jY_0\mu_{12}$$
$$c_4 = jY_0(\mu_{12} + \mu_{22})$$

The realized network has only $4N$ components

Other Design Methods

Other Design Methods that Selectively Eliminate Components



- Figures (a), (b) and (c) show three other topologies that may be derived
- All three methods are "'suboptimal", since they require N more components than Minimum-Complexity Method
- They may have other advantages (such as high bandwidth)

Overview

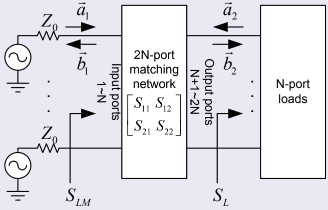
Definition of Bandwidth

- Bandwidth of a single-port matched load is the frequency range for which less than 4% power is reflected

Power Reflection Ratio

The *power reflection ratio* r in the RF system is the ratio between the expected reflected power and the expected incident power

$$r = \frac{\mathbb{E}\mathrm{tr}\{\vec{b}_1^H \vec{b}_1\}}{\mathbb{E}\mathrm{tr}\{\vec{a}_1^H \vec{a}_1\}} = \frac{\mathbb{E}\mathrm{tr}\{\vec{a}_1^H S_{LM}^H S_{LM} \vec{a}_1\}}{\mathbb{E}\mathrm{tr}\{\vec{a}_1^H \vec{a}_1\}} = \frac{1}{N} \|S_{LM}\|_F^2$$



Bandwidth

The *bandwidth* of the N matched loads is the frequency range that $r \leq 0.04$ in the vicinity of the design frequency f_d.

$$f_{\mathrm{BW}} = \max\{f_2 - f_1 : f_1 \leq f_d \leq f_2, r(f) \leq 0.04, \forall f_1 \leq f \leq f_2\}$$

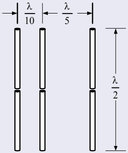
where f_1, f_2 are lower and upper cutoff frequency, respectively.

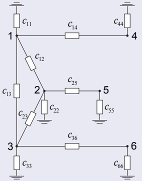
Bandwidth Analysis of Decoupling Networks

- The admittance of the decoupling network at frequency $f = f_d + \Delta f$ is
$$Y(f) = Y + Y_e(\Delta f) \approx Y + Y_f \Delta f$$
- When the decoupling network is connected to the loads, the first order r and bandwidth are
$$r \approx \frac{\Delta f^2}{4N} \left\| \begin{bmatrix} I & VA^H \end{bmatrix} Y_f \begin{bmatrix} I \\ A^* V^T \end{bmatrix} \right\|_F^2$$
$$f_{\text{BW}} \approx f_{\text{BW}}^{(1)} = \frac{0.8\sqrt{N}}{\left\| \begin{bmatrix} I & VA^H \end{bmatrix} Y_f \begin{bmatrix} I \\ A^* V^T \end{bmatrix} \right\|_F}$$
- When Π-network is used
$$[Y_f]_{ij} = \begin{cases} -|y_{ij}| & i \neq j \\ |\sum_{k=1}^{2N} y_{ik}| + \sum_{k=1,k\neq i}^{2N} |y_{ik}| & i = j \end{cases}$$
- Optimization problem
$$\max_{V:V^H V = I} f_{\text{BW}}^{(1)}$$

High-Bandwidth Decoupling Network Design

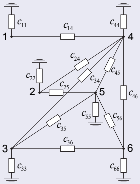
Comparison of Two Design Methods (at 2.4 GHz)





c_{11}	32.06 pF
c_{12}	9.20 pF
c_{13}	2.24 nH
c_{14}	29.66 pF
c_{22}	3.46 pF
c_{23}	4.26 pF
c_{25}	3.96 pF
c_{33}	3.13 pF
c_{36}	5.41 pF
c_{44}	0.29 nH
c_{55}	2.12 nH
c_{66}	2.14 nH

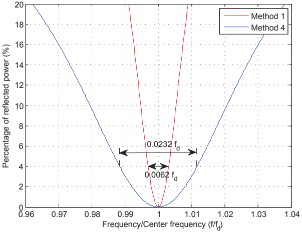
Minimum Complexity



c_{11}	7.78 nH
c_{14}	0.57 pF
c_{22}	7.28 nH
c_{24}	0.34 pF
c_{25}	0.26 pF
c_{33}	4.71 nH
c_{34}	0.045 pF
c_{35}	0.16 pF
c_{36}	0.73 pF
c_{44}	5.38 nH
c_{45}	0.79 pF
c_{46}	156.51 nH
c_{55}	7.25 nH
c_{56}	0.42 pF
c_{66}	6.82 nH

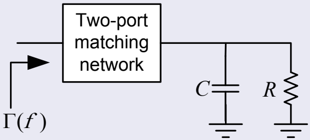
High Bandwidth

- High Bandwidth Method has three times the bandwidth of Minimum-Complexity Method
- What is the best we can do?
- Increase the bandwidth by cascading multi-port networks?



Theoretical Bounds on Bandwidth

Bode-Fano Bound



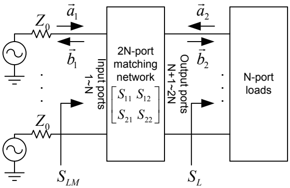
- Well-known Bode-Fano result for an RC load: Theoretical limit on bandwidth

$$\int_0^\infty \ln \frac{1}{|\Gamma(f)|} df \leq \frac{\pi}{RC}$$

- Find a high-dimensional version of the Bode-Fano bound?

Lossy Matching Networks

- For lossy matching networks, the total power delivered to the loads matters



Definition for Power Delivery Ratio

The *power delivery ratio d* is the ratio between the expected power delivered to the loads and the expected incident power

$$d = \frac{\mathbb{E}\mathrm{tr}\{\vec{b}_2^H \vec{b}_2 - \vec{a}_2^H \vec{a}_2\}}{\mathbb{E}\mathrm{tr}\{\vec{a}^H \vec{a}\}} = \frac{1}{N}\mathrm{tr}\{S_{21}^H (I - S_{22}S_L)^{-H}(I - S_L^H S_L)(I - S_{22}S_L)^{-1}S_{21}\}$$

- Find the decoupling network that is least sensitive to the resistive losses?
- Design lossy matching network that maximizes d?

Overview

Summary

- What we showed:
  - Systematic design of decoupling networks using $N^2 + N$ components, the minimum possible
  - Decoupling networks realization using linear number of components for special loads
  - High-bandwidth designs (that are not minimum complexity)
- Rich area for future work:
  - What is best we can do in bandwidth?
  - How do we handle lossy components?
  - How do we layout complicated decoupling networks?
  - What load properties lead to linear-in-N complexity networks?

References

D. Nie, B. Hochwald, E. Stauffer, "Systematic design of large-scale multiport decoupling networks," *IEEE Transactions of Circuits and Systems I: Regular Papers*, vol. 61, July 2014, pp. 2172–2181.

S. Blanch, J. Romeu, I. Corbella, "Exact representation of antenna system diversity performance from input parameter description," *Electronic Letters*, vol. 39, May 1, 2003, pp. 705–707.