ARQ Protocols and Sphere Decoding in MIMO Systems

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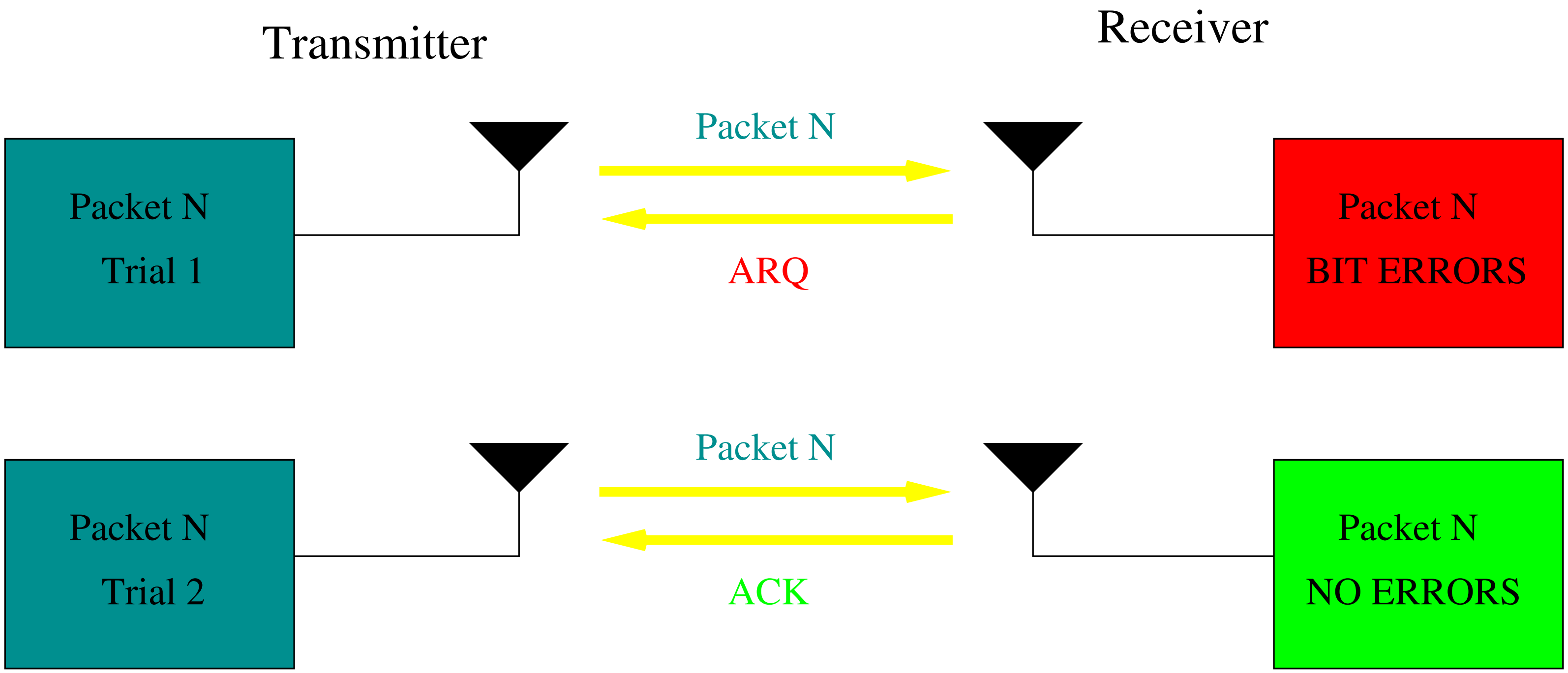
Automatic Repeat ReQuest (ARQ) in Error Protection

Two types of error protection: forward error correction (FEC) coding, ARQ.

ARQ: If a received packet contains any bit errors, the receiver requests a retransmission of the packet (known as selective-repeat ARQ).



Overview

Look to enhance integration of MIMO with ARQ is available by proposing two techniques:

- adapting the symbol mappings (mapping diversity) used in retransmissions, and
- combining retransmissions using joint (ML) decoding at the receiver.

Problem bears many similarities to space-time block coding (STBC). STBC uses a predetermined code rate (redundancy) while ARQ uses incremental redundancy with feedback control.

Background

Onggosanusi et al. [2003] introduced packet transmission combining using zeroforcing and and MMSE receivers.

Ding/Rice [2003] proposed hybrid ARQ involving spatio-temporal vector coding and multi-D TCM.

Nguyen/Ingram [2001] studied hybrid ARQ for systems with recursive space-time codes.

Zheng et al. [2002] suggested dividing ARQ into ARQ subprocesses that operate over isolated pairs of transceiver antennas.

Effects of bit-to-symbol relationships have not been considered.

Symbol Mapping Diversity

Simple scheme for bandwidth-efficient modulations such as PSK and QAM which using different bit-to-symbol mappings for retransmissions [Samra and Ding 2003].

Consecutive groups of $\log_2 |\mathcal{C}|$ bits (referred to as labels) are assigned to symbols in constellation $\mathcal{C}$ via a symbol mapping function $\psi : \{0, 1, \ldots, |\mathcal{C}| - 1\} \to \mathcal{C}$.

Remapping essentially equalizes the Euclidean distances between labels (i.e. closely-spaced labels in initial transmission are spaced far apart in retransmission).

Mappings separately designed for AWGN and flat-fading channels.

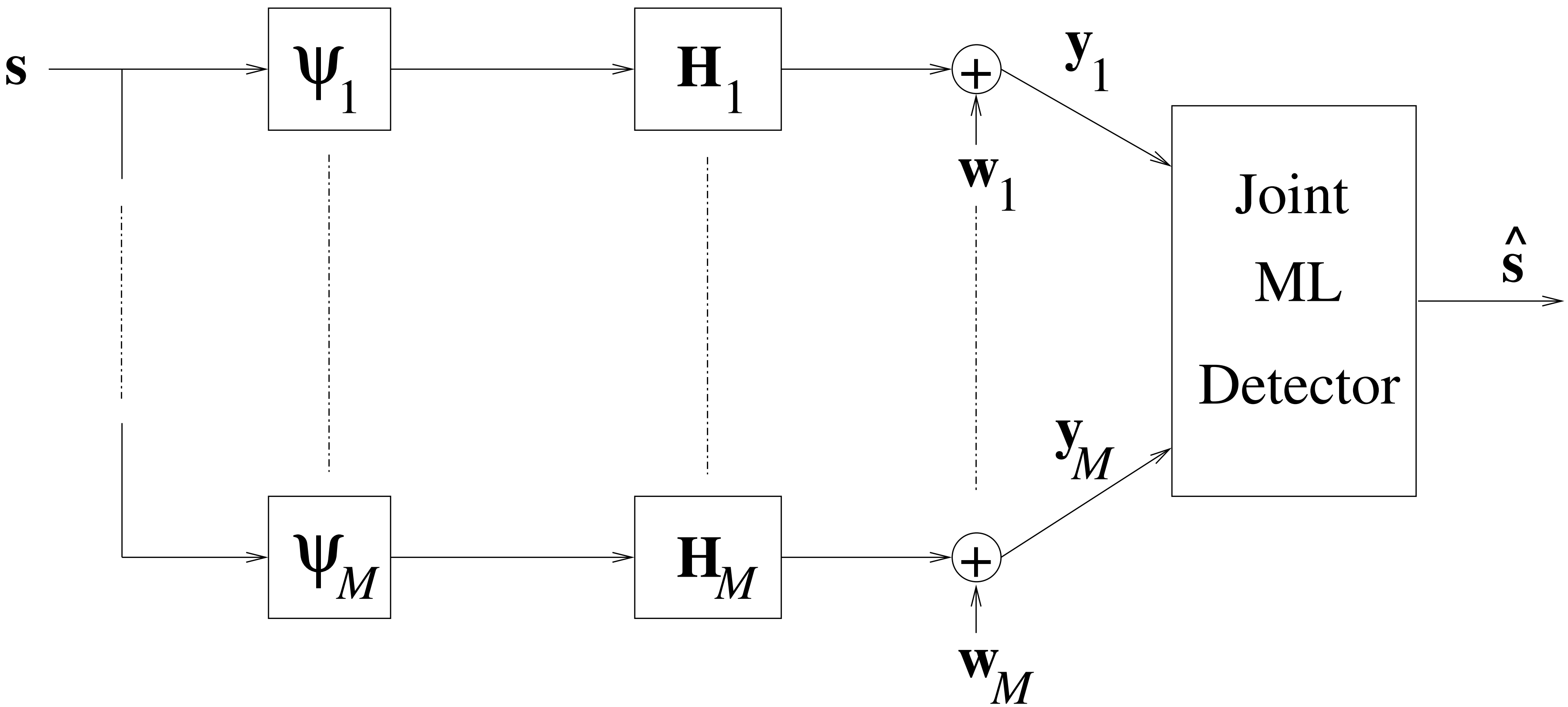
MIMO Systems and ARQ

Consider M transmissions of a block of N labels $\mathbf{s} = [s_1, \ldots, s_N]^T \in \mathcal{C}^N$.

For the m^{th} transmission of $\mathbf{s}$, the channel $\mathbf{H}_m$ is A $K \times N$ matrix, with $h_{m,ik}$ indicating the Rayleigh fading coefficient between transmit antenna i and receive antenna k

Each label s_n is distinctly mapped via M mapping functions $\psi_1, \ldots, \psi_M$.

Problem Model



Problem Model (cont.)

Receiver obtains:

$$\mathbf{y}_m = \begin{bmatrix} y_{m,1} \\ \vdots \\ y_{m,K} \end{bmatrix} = \mathbf{H}_m \begin{bmatrix} \psi_m[s_1] \\ \vdots \\ \psi_m[s_N] \end{bmatrix} + \begin{bmatrix} w_{m,1} \\ \vdots \\ w_{m,K} \end{bmatrix} \quad (1)$$

$$= \mathbf{H_m}\vec{\psi}_m[\mathbf{s}] + \mathbf{w}_m.$$

Receiver can employ a joint ML decoding

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \sum_{m=1}^{M} ||\mathbf{y}_m - \mathbf{H}_m \vec{\psi}_m[\mathbf{s}]||^2.$$

Low cost approximations are needed (sphere decoding).

Applying Sphere Decoding

Employ sphere decoding to reduce complexity from $|\mathcal{C}|^N$ to roughly N^3 [Agrell 2002, Hassibi 2002, Ten Brink 2003].

Define $\mathbf{p}_m = \mathbf{H}_m^\dagger \mathbf{y}_m$ and upper-triangular matrix $\mathbf{U}_m$ so that $\mathbf{U}_m^H \mathbf{U}_m = \mathbf{H}_m^H \mathbf{H}_m$.

Metric for minimization becomes:

$$\sum_{n=1}^{N} \sum_{m=1}^{M} u_{m,nn}^2 \left| \psi_m[s_n] - p_{m,n} + \sum_{k=n+1}^{N} \frac{u_{m,nk}}{u_{m,nn}} (\psi_m[s_k] - p_{m,k}) \right|^2$$

Applying Sphere Decoding (cont.)

Define a hypersphere of radius R centered by $\mathbf{p}_1, \ldots, \mathbf{p}_M$. Iteratively select estimates $\hat{s}_N, \ldots, \hat{s}_1$ within hypersphere.

Select the estimate $\hat{s}_n$ from the set $\mathcal{S}_n$ of labels that fall inside the hyper-ellipsoid region $\mathcal{E}_n$ defined by

$$\sum_{m=1}^{M} u_{m,nn}^2 \left|\psi_m[s_n] - a_m\right|^2 < r_n^2,$$

with a_m and r_n computed from the existing estimates of $\hat{s}_{n+1}, \ldots, \hat{s}_N$:

Applying Sphere Decoding (cont.)

$$a_m = p_{m,n} - \sum_{k=n+1}^{N} \frac{u_{m,nk}}{u_{m,nn}} (\psi_m[\hat{s}_k] - p_{m,k})$$

$$b_m = \sum_{k=n+1}^{N} u_{m,kk}^2 \left| \psi_m[\hat{s}_k] - p_{m,k} + \sum_{t=k+1}^{N} \frac{u_{m,kt}}{u_{m,kk}} (\psi_m[\hat{s}_t] - p_{m,t}) \right|^2 ,$$

$$r_n^2 = R^2 - \sum_{m=1}^{M} b_m .$$

Applying Sphere Decoding (cont.)

If no label $\hat{s}_n$ existing within this region, we invalidate the estimate $\hat{s}_{n+1}$, and choose a new estimate from $\mathcal{S}_{n+1}$. If $\mathcal{S}_{n+1}$ is empty, we retreat to $\mathcal{S}_{n+2}$, etc.

When $\hat{s}_1$ is chosen, its distance becomes the new R, and process is repeated to find better estimates.

Key to fast performance is quick enumeration of candidates within $\mathcal{E}_n$.

Enumerating Candidates

When $M = 1$, techniques are readily available [Hochwald and Ten Brink 2003]. They do not easily extend for $M > 1$.

In computational vision research, problems frequently require closest point searches in high-dimensional spaces [Nene and Nayar 1996].

Define a hyper-box $\mathcal{D}_n$ that tightly bounds $\mathcal{E}_n$.

Enumerating Candidates (cont.)

Presort all labels in $\mathcal{C}$ along each of the $2M$ dimensions, easily determine candidates inside $\mathcal{D}_n$ using binary searches.

Simply evaluate all labels in $\mathcal{D}_n$ to find those in $\mathcal{E}_n$, or treat $\mathcal{D}_n$ as $\mathcal{E}_n$ (only effective for small M).

As M increases, likelihood of large $u_{m,nn}$ and/or large a_m increases as well, meaning that $\mathcal{D}_n$ and $\mathcal{E}_n$ will contain fewer labels.

Using Mapping Diversity

Channel is memoryless, but N labels simultaneously interfere.

Sphere decoding suggests that each label can be treated individually, viewing $u_{m,nn}$ as a fading gain.

Use existing mappings for flat-fading channels.

Mapping diversity should also lead to a sparser distribution of labels of the $2M$ dimensions, and $\mathcal{E}_n$ may contain fewer labels.

Precoding for Static Channels

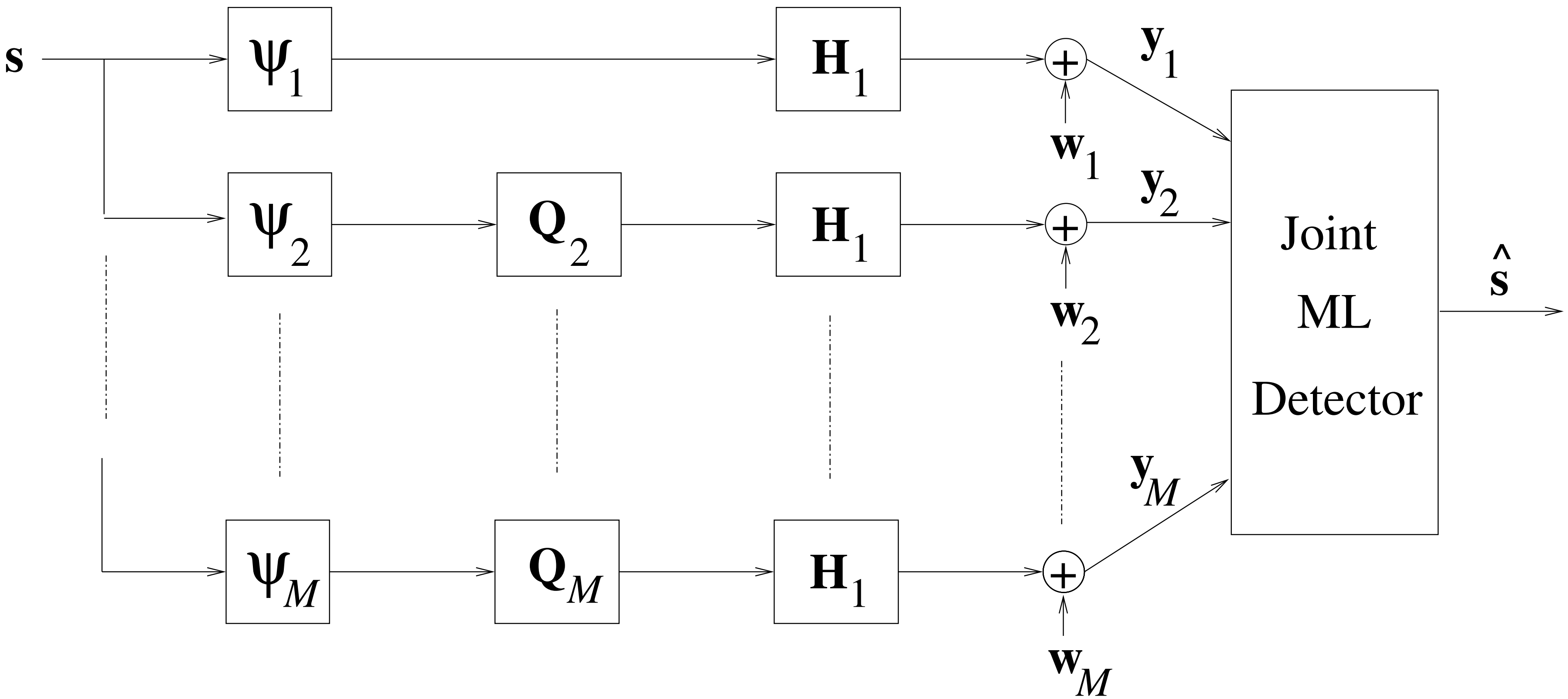
Variation among these channels $\mathbf{H}_1, \ldots, \mathbf{H}_M$ provides a stronger diversity effect, and better performance.

When $\mathbf{H}_1 = \mathbf{H}_2 = \cdots = \mathbf{H}_M$, we precode each transmission using an $N \times N$ matrix $\mathbf{Q}_m$.

A trace constraint, $\mathrm{tr}\{\mathbf{Q}_m \mathbf{Q}_m^H\} = N$, is necessary to maintain the transmitted signal power.

Design with no knowledge of the channel.

Precoding for Static Channels (cont.)



Precoding for Static Channels (cont.)

Permutation matrix simply shuffles the label-transmit antenna assignments for each transmission.

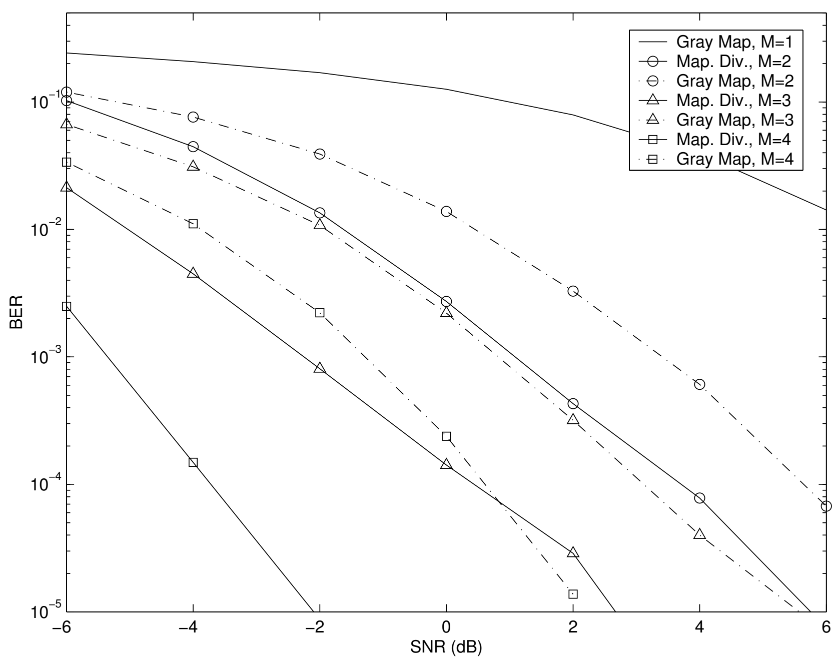
Second option is an FFT which spreads the symbol energy evenly among the N transmit antennas so that the effects of any deep fades (i.e. small values in $\mathbf{H}_m$) are alleviated.

Suggest constructing $\mathbf{Q}_m$ by uniquely permuting the rows and columns of an FFT matrix.

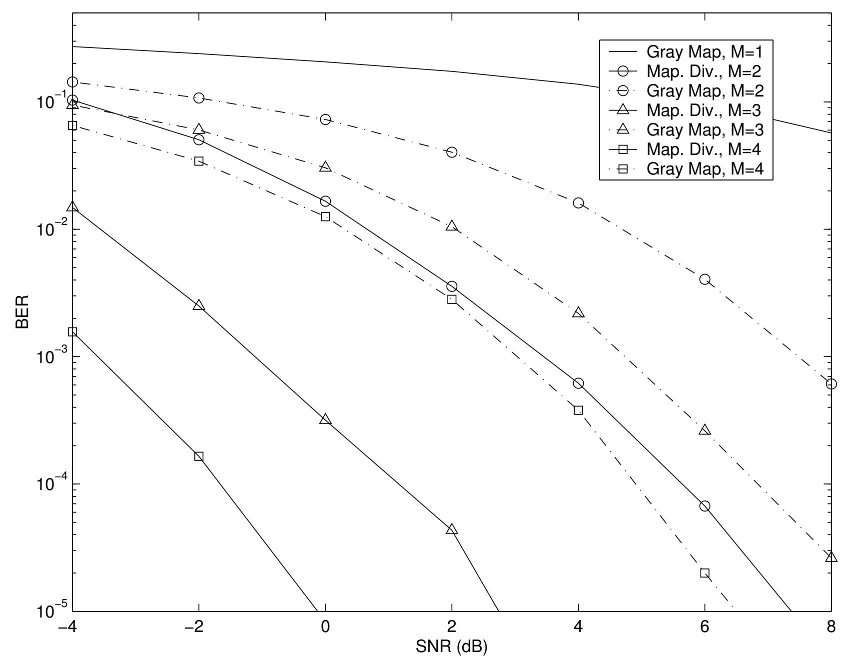
ARQ Simulation Results

100000 symbol vectors, $K = 4$, $N = 4$, channel variation

16QAM



64QAM

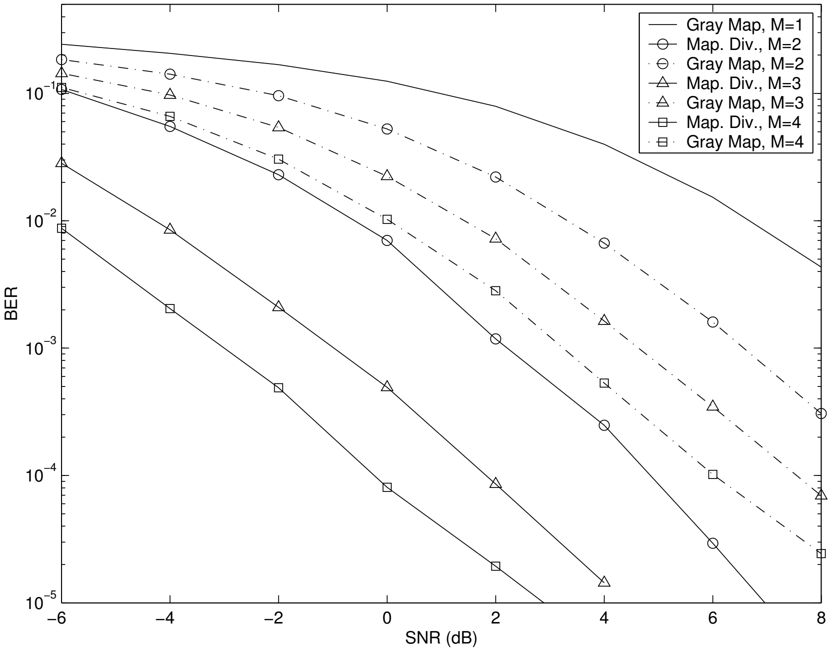


ARQ Simulation Results (cont.)

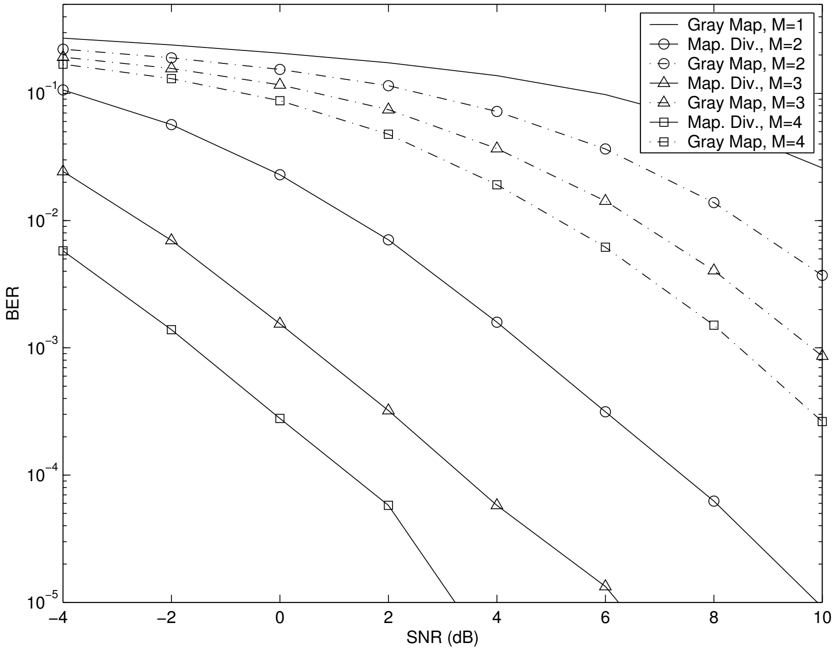
$K = 4$ (Rx antennas), $N = 4$ (Tx antennas), 4×4 static channels

2000 packets of Monte Carlo simation with packet size of 800 bits.

16QAM



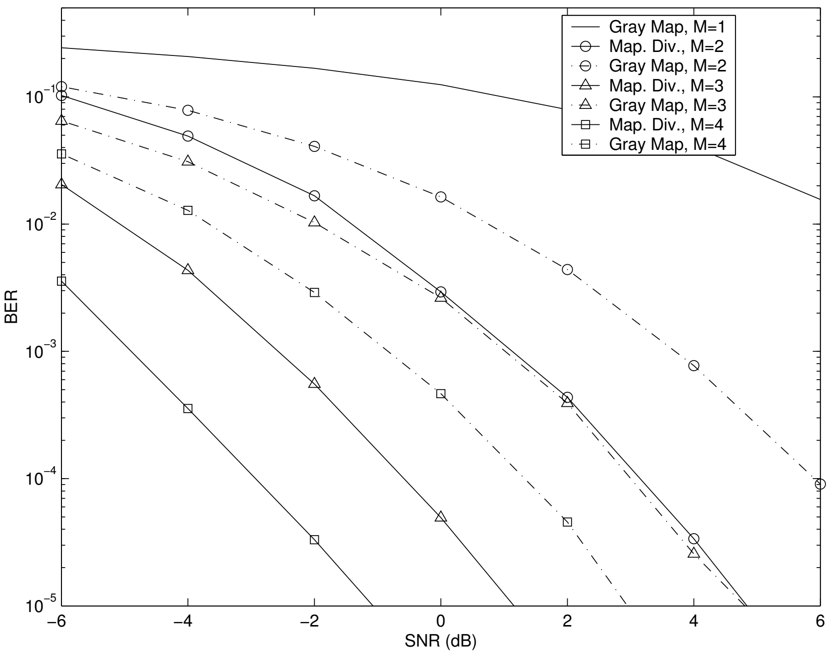
64QAM



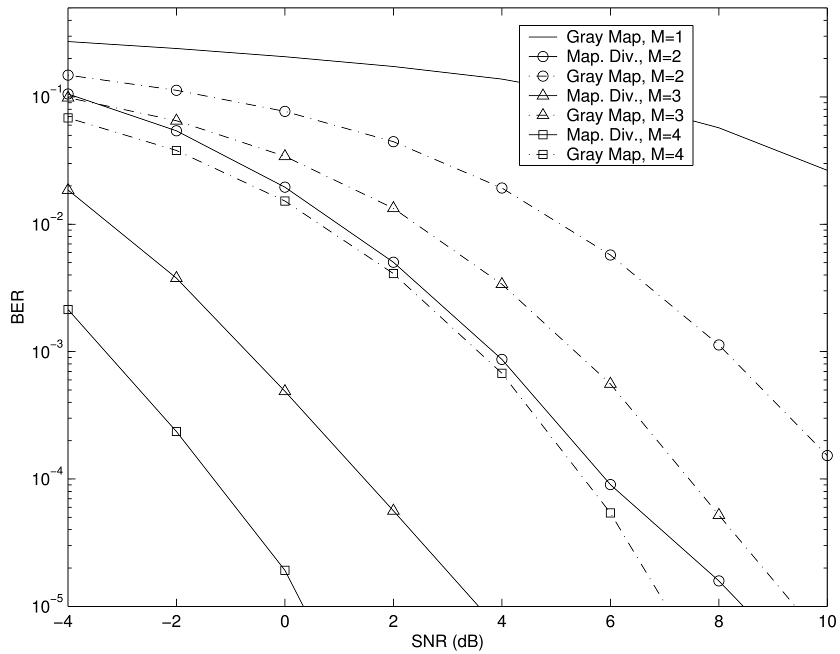
ARQ Simulation Results (cont.)

100000 symbol vectors, $K = 4$, $N = 4$, static channels w/ precoding

16QAM



64QAM



ARQ Simulation Results (comparison)

Mapping diversity provides significant gains in BER.

With independent channels ($M = 2$), 2 dB gain (16QAM) and over a 4 dB gain (64QAM).

With identical channels ($M = 2$), 4 dB gain (16QAM) and over a 7 dB gain (64QAM).

Retransmission precoding transforms identical channels into diversity channels. channels.

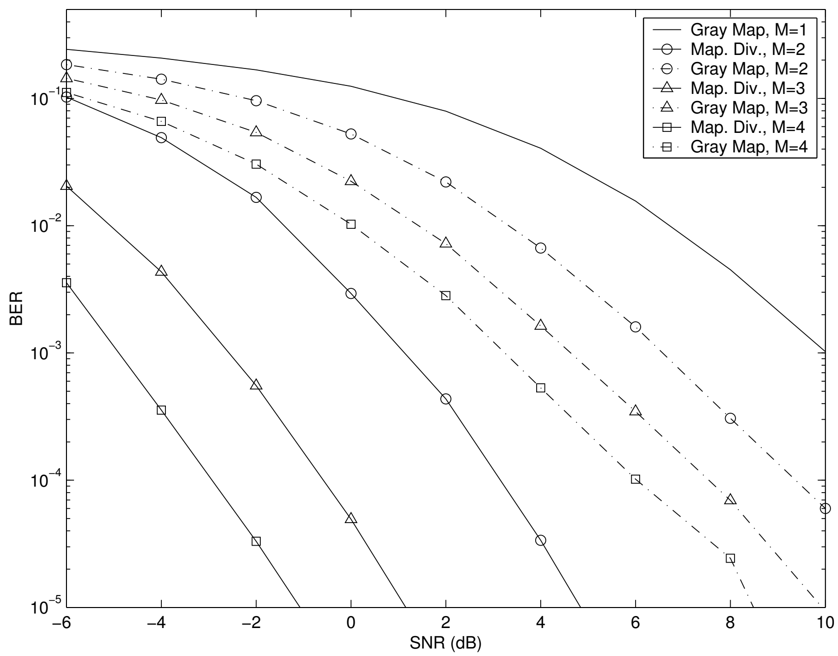
As a space-time code, precoded mapping diversity results can be comparied against a simple repetition code. $M = 4$ implies full-rate STC. The coding gains are approximately 8dB (16QAM) and 12dB (64QAM).

STC Comparison Results

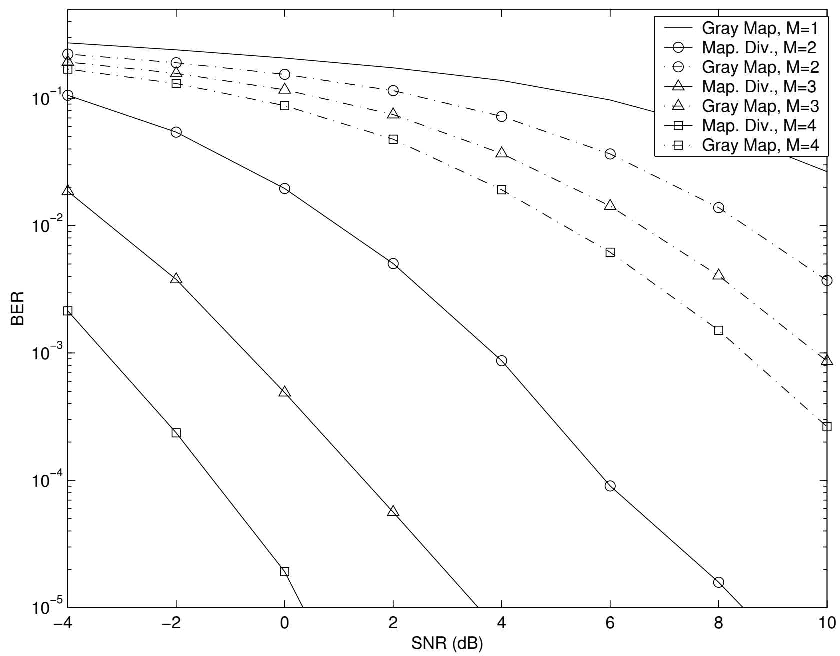
100000 symbol vectors, $K = 4$, $N = 4$, static channel

Compare Gray mapping with no precoding against mapping diversity with precoding.

16QAM



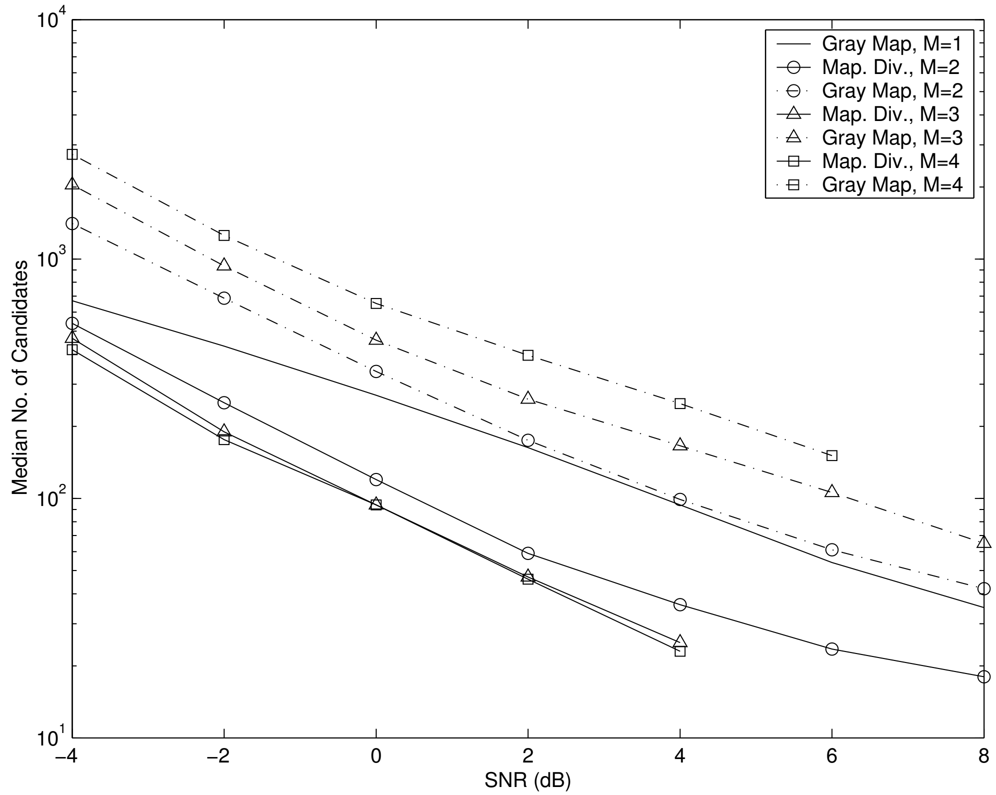
64QAM



Complexity Results Results

4×4 64QAM w/ channel variation. Complexity: med. No. of candidates.

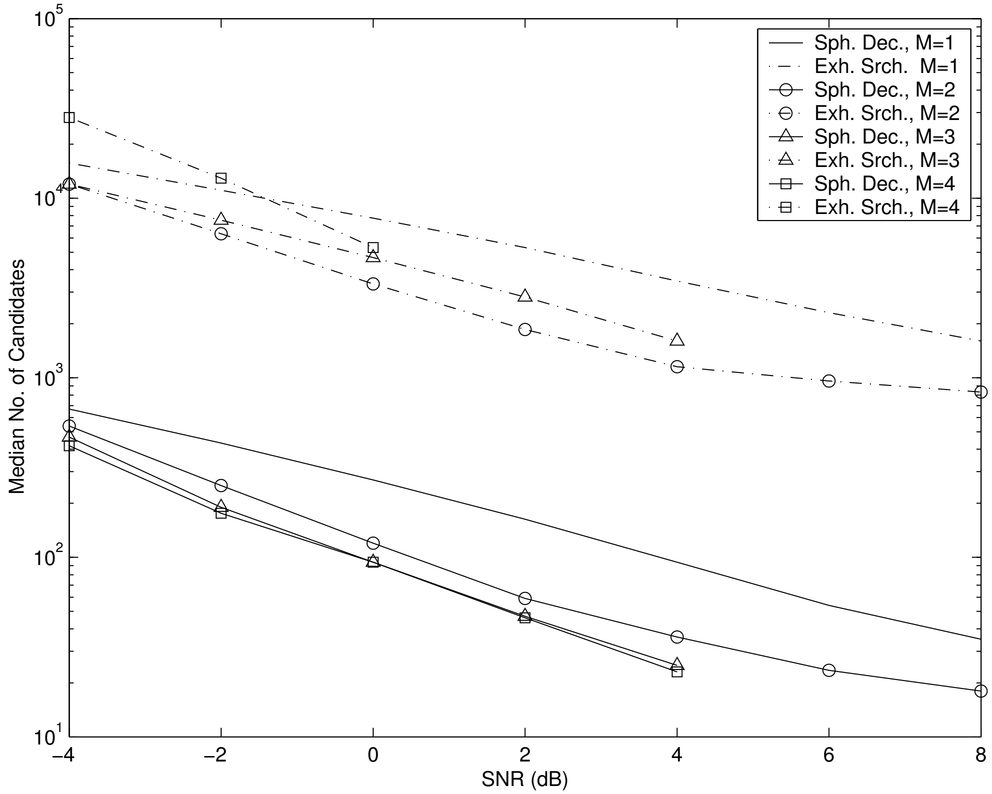
Gray repetition vs. optimal precoded mapping



Complexity Results Results

4×4 64QAM w/ channel variation. Complexity: med. No. of candidates.

Exhaustive search vs. Proposed search



Conclusion

An ARQ protocol for MIMO is proposed.

Significant gain shown from exploiting mapping diversity.

Precoding can be easily integrated.

Joint receiver can apply sphere decoding and fast enumeration.

System requires no major hardware or standard change.