

## Transmitting over *III*-behaved MIMO Channels

Smart Antenna Workshop

Stanford University

July 2004

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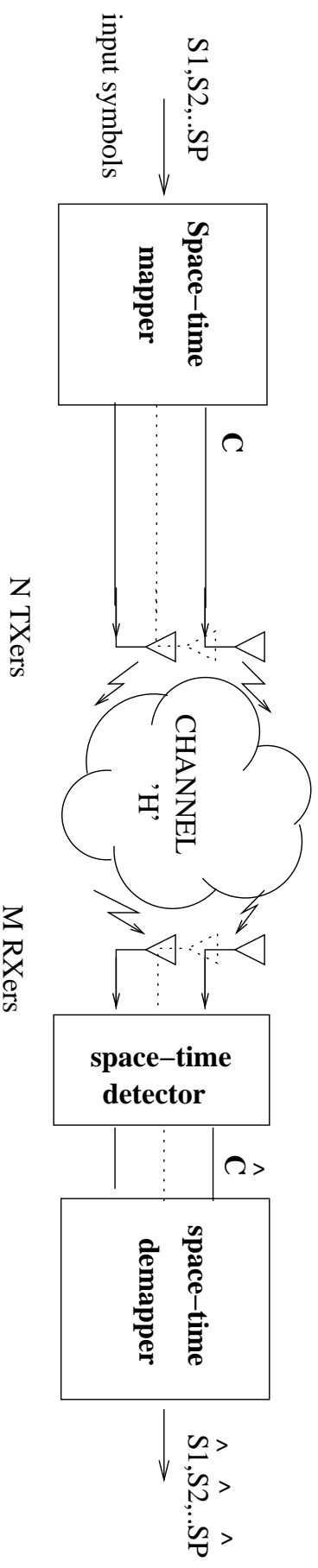
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# Outline

- Space-time codes
- Ill-behaved MIMO channels
- A Precoding approach
  - For low-rate codes
  - For high-rate codes

# Space-time codes



## Space-time codes: General model

Step 1:  $P$  input (QAM) modulation symbols are mapped to a codeword matrix of size  $M_t \times K$ .

$$\{s_1, s_2, \dots, s_P\} \Rightarrow \mathbf{C} = \begin{bmatrix} c_1(1) & c_1(2) & \dots & c_1(K) \\ \vdots & \vdots & \ddots & \vdots \\ c_{M_t}(1) & c_{M_t}(2) & \dots & c_{M_t}(K) \end{bmatrix}$$

Step 2: The codeword is launched into the channel. We receive:

$$\mathbf{Y} = \begin{bmatrix} h_{11} & \dots & h_{1M_t} \\ \vdots & & \vdots \\ h_{M_r,1} & h_{M_r,2} & h_{M_r,M_t} \end{bmatrix} \begin{bmatrix} c_1(1) & c_1(2) & \dots & c_1(K) \\ \vdots & \vdots & \ddots & \vdots \\ c_{M_t}(1) & c_{M_t}(2) & \dots & c_{M_t}(K) \end{bmatrix} + \text{Noise}.$$

## Low rate vs. high rate codes

### Low rate codes

- Diversity (redundancy) oriented: E.g.  $P = K$  (rate 1 symbol per channel use).
- Orthogonal ST codes are such that  $C$  is a unitary matrix (up to a scalar).
- For uncorrelated channels: Diversity order is  $M_t M_r$ .
- **Sensitive to correlation. Moderately sensitive to rank.**

### High rate codes

- Rate (low redundancy) oriented: E.g.  $P = K M_t$  (rate  $M_t$  symbols per channel use).
- **Sensitive to correlation. Very sensitive to rank.**
- **Unable** to extract capacity in strongly Ricean or correlated channels.

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## Ill-behaved MIMO channels

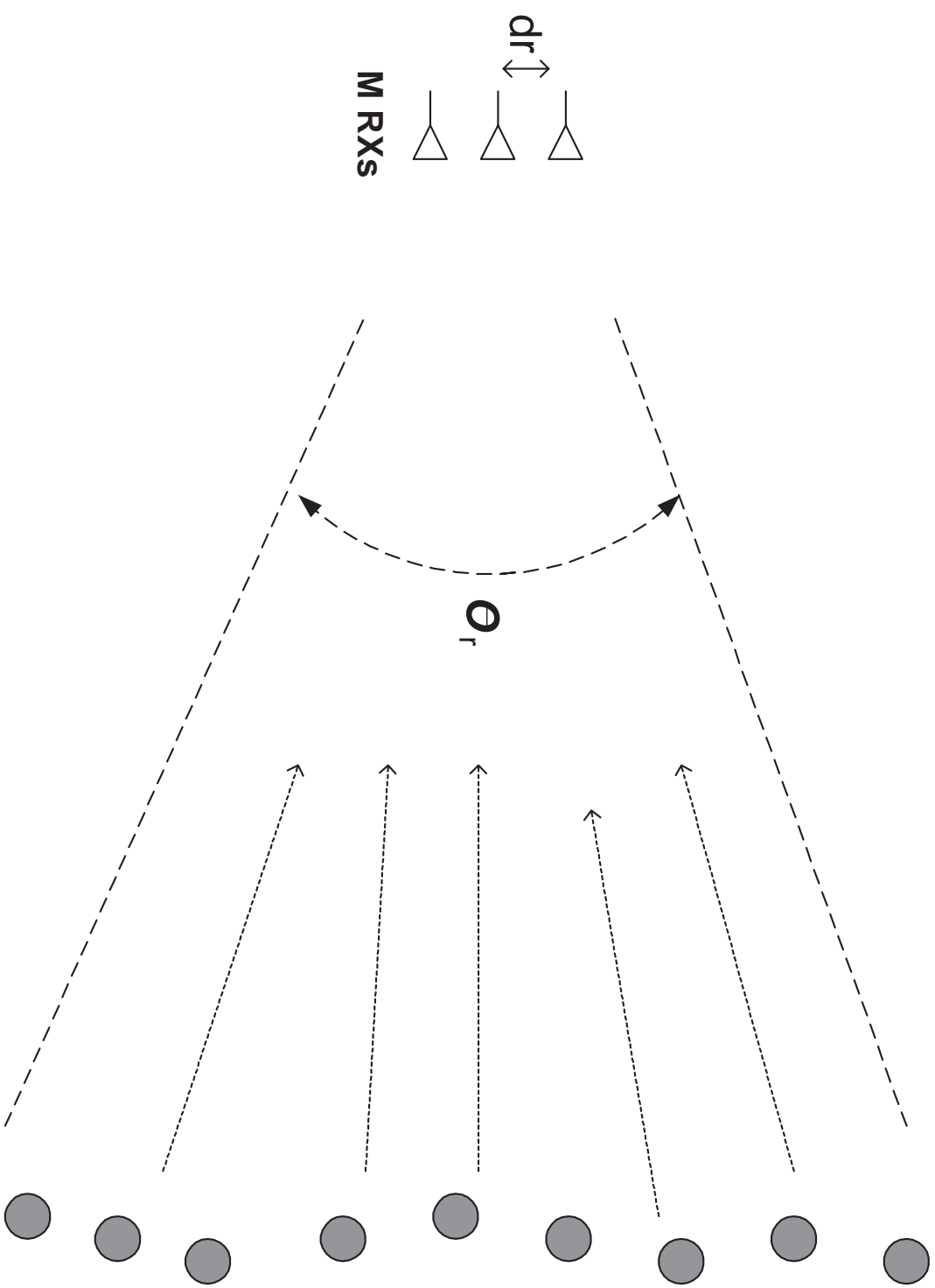
Ill-behaved channels occur due to:

- Fading correlation at TXer and/or RXer
- Correlation model is non separable (Kronecker model not realized)
- Low rank Rice component dominates

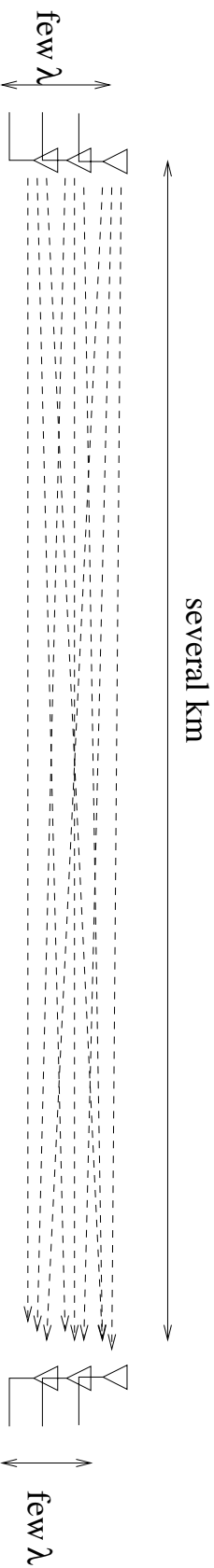
Leading to:

- *Effective diversity order reduced*
- *Effective rank reduced*

# Antenna Array Correlation



## Rank behavior of Rice component



The rank of LOS component is full iff [Gesbert et al. TCOM Dec. 2002]

$$A_t A_r \geq \frac{4\lambda R}{M_r}$$

where  $A_t$ , (resp.  $A_r$ ) is TX (resp. RX) array aperture,  $R$  is range...**never realized!**



## Dealing with ill-behaved MIMO channels

- Propagation (Rice, correlation) scenario hard to predict
- Transmitter design must be *adaptive*.
- Low-rate statistical feedback is realistic.
- We want to keep rate constant.
- We want low-complexity closed-form solution where possible!

### Idea

- **Design a linear precoder based on statistical feedback**
  - Part 1: for low-rate space-time block codes
  - Part 2: for high-rate space-time (multiplexing) codes

## Part one

- **Linear precoding for low-rate space-time orthogonal block codes**

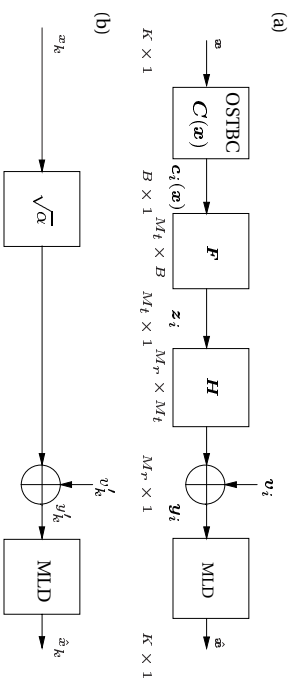
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## The interesting questions!

Let us assume the TX knows the full correlation information (TX,RX) but not the instantaneous channel...

1. Can we tackle transmitter correlation using a linear precoder?
  - Answer: **Yes!** [Giannakis et al][Sampath et al][others]
2. Can we find the optimal precoder in minimum SER sense?
  - Answer: **Unknown!** (Only bounds on pairwise error prob. used so far)
3. Can we find the precoder for correlated receiver case? Is there any gain?
  - Answer: **Unknown!** (Ergodic capacity says no gain!)
4. Can we find the precoder for non-separable TX-RX correlations?
  - Answer: **Unknown!**
5. Can we find a good precoder in closed-form for correlated receiver case?
  - Answer: **Unknown!**

# Precoded Space-time Block Coding



$$Y = HFC(s) + V. \quad (1)$$

## Correlated MIMO channels

General case: Joint correlation at TX and RX

$$\text{vec}(\mathbf{H}) = \mathbf{R}^{1/2} \text{vec}(\mathbf{H}_w) \quad (2)$$

where  $\mathbf{H}_w$  is i.i.d. Rayleigh MIMO matrix

The Kronecker case (Correlation at TX and RX are separable)

$$\mathbf{R} = E [\text{vec}(\mathbf{H}) \text{vec}^H(\mathbf{H})] = \mathbf{R}_t^T \otimes \mathbf{R}_r, \quad (3)$$

...is not always true! [Bonek et al. 04]

## Expressions for Symbol Error Rate (SER)

For M-QAM (also available for M-PSK, M-PAM [Simon & Alouini]):

$\gamma$  is the instantaneous SNR, depending on channel, precoder

$$SER_{\gamma} = \frac{4}{\pi} \left( 1 - \frac{1}{\sqrt{M}} \right) \left[ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{-\frac{g_{QAM}\gamma}{\sin^2(\theta)}} d\theta + \frac{1}{\sqrt{M}} \int_0^{\frac{\pi}{4}} e^{-\frac{g_{QAM}\gamma}{\sin^2(\theta)}} d\theta \right], \quad (4)$$

Average SER:

$$SER \triangleq \Pr\{\text{Error}\} = \int_0^{\infty} SER_{\gamma} p_{\gamma}(\gamma) d\gamma. \quad (5)$$

Looks like the definition of a moment generating function  $\phi_{\gamma}(s) = \int_0^{\infty} p_{\gamma}(\gamma) e^{s\gamma} d\gamma$ .

## Exact SER as function of precoder

For M-QAM (also available for M-PSK, M-PAM) [Hjorungnes & Gesbert 04]:

$$SER = \frac{4\sqrt{M}-1}{\pi\sqrt{M}} \left[ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{d\theta}{\det\left(\mathbf{I}_{M_t M_r} + \frac{\delta_{\text{QAM}}^g}{\sin^2\theta} \Phi\right)} + \frac{1}{\sqrt{M}} \int_0^{\frac{\pi}{4}} \frac{d\theta}{\det\left(\mathbf{I}_{M_t M_r} + \frac{\delta_{\text{QAM}}^g}{\sin^2\theta} \Phi\right)} \right] \quad (6)$$

Where matrix  $\Phi$  defined by

$$\Phi = \mathbf{R}^{1/2} \left[ (\mathbf{F}^* \mathbf{F}^T) \otimes \mathbf{I}_{M_r} \right] \mathbf{R}^{1/2}. \quad (7)$$

# The optimum precoder: problems and properties

Optimum precoder:

$$\min_{\{\mathbf{F} \in \mathbb{C}^{M_t \times B}\}} SER \quad (8)$$

$$\text{subject to } K \alpha \sigma_x^2 \text{Tr} \{ \mathbf{F} \mathbf{F}^H \} = P \quad (9)$$

Some properties:

- If  $\mathbf{F}$  is an optimal precoder, then  $\mathbf{F}\mathbf{U}$ , where  $\mathbf{U}$  is unitary, is also optimal.
- if  $\text{SNR} \rightarrow \infty$  then the optimal precoder is given by the trivial precoder  $\mathbf{F} = \beta \mathbf{I}_{M_t}$  for the  $M$ -PSK,  $M$ -PAM, and  $M$ -QAM constellations.
- If  $\mathbf{R} = \mathbf{I}_{M_t M_r}$ , then the optimal precoder is given by the trivial precoder  $\mathbf{F} = \beta \mathbf{I}_{M_t}$  for the  $M$ -PSK,  $M$ -PAM, and  $M$ -QAM constellations.
- If Kronecker model holds and  $\mathbf{R}_t = \mathbf{I}_{M_t}$ , then  $\mathbf{F} = \beta \mathbf{I}_{M_t}$ .



## One more property

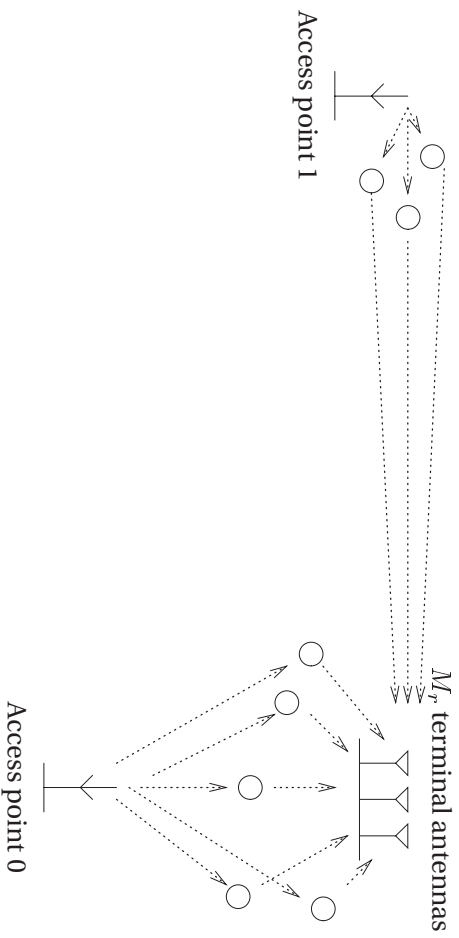
If no transmitter correlation, then the general-case correlation writes:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{r_0} & \mathbf{0}_{M_r \times M_r} & \cdots & \mathbf{0}_{M_r \times M_r} \\ \mathbf{0}_{M_r \times M_r} & \mathbf{R}_{r_1} & \cdots & \mathbf{0}_{M_r \times M_r} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{M_r \times M_r} & \mathbf{0}_{M_r \times M_r} & \cdots & \mathbf{R}_{r_{M_f-1}} \end{bmatrix}, \quad (10)$$

Property:

- In this case, optimal  $\mathbf{F}$  can be chosen diagonal, real, positive, up to a unitary matrix. (power scaling strategy).

# Non separable-No TX correlation



# Non separable-No TX correlation: A closed form solution

Equivalent SISO channel formulation for  $M_t = 2$  case:

$$\alpha = f_0^2 \|\mathbf{h}_0\|^2 + f_1^2 \|\mathbf{h}_1\|^2 = f_0^2 \sum_{j=0}^{M_r-1} \lambda_{r_0,j} |h'_{w_0,j}|^2 + f_1^2 \sum_{j=0}^{M_r-1} \lambda_{r_1,j} |h'_{w_1,j}|^2. \quad (11)$$

where  $\{\lambda_{r_0,j}\}$  and  $\{\lambda_{r_1,j}\}$  are the eigenvalues of  $\mathbf{R}_{r_0}$  and  $\mathbf{R}_{r_1}$ .  $h_{w_i,j}$  are i.i.d. Gaussian.

**Maximum diversity spread principle:** *The symbol energy should be spread as much as possible over all independent components of the equivalent SISO channel.*

# The Maximum Diversity Spread Solution

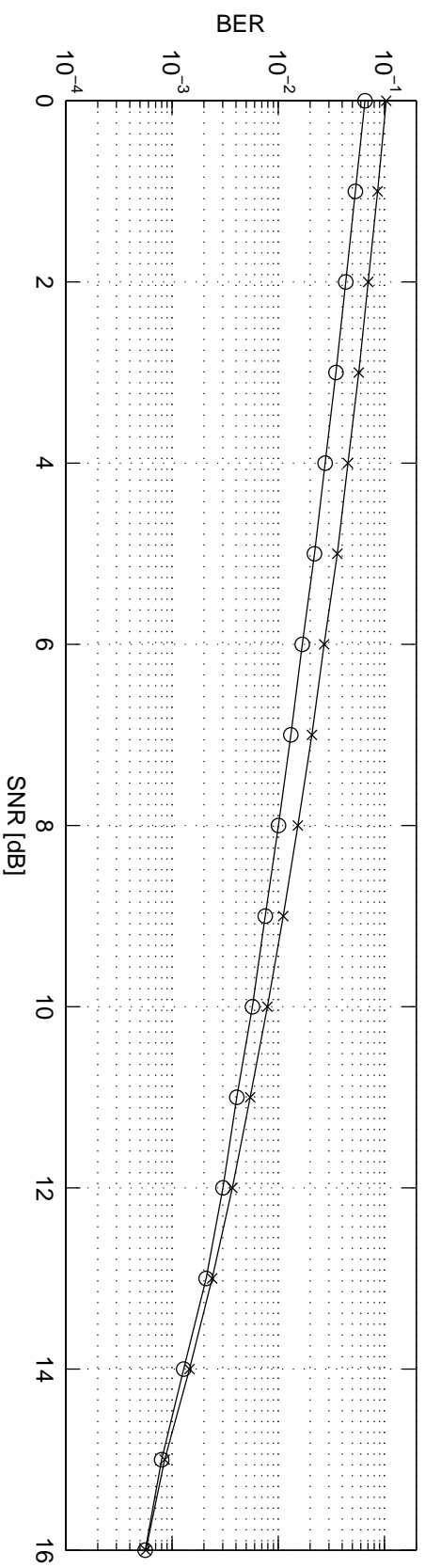
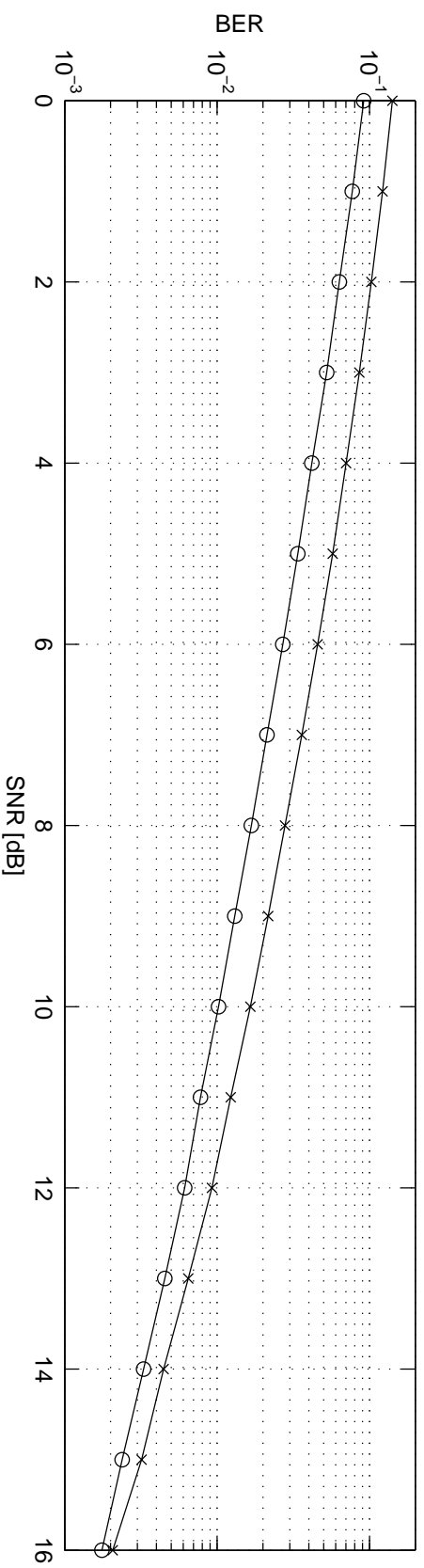
$$\min_{f_0, f_1 \geq 0} \sum_{i=0}^1 \sum_{j=0}^{M_r-1} \left( f_i^2 \lambda_{r_{ij}} - \frac{1}{2M_r} \sum_{i=0}^1 \sum_{j=0}^{M_r-1} f_i^2 \lambda_{r_{ij}} \right)^2 \quad \text{subject to } f_0^2 + f_1^2 = 1, \quad (12)$$

With parameterization  $f_0 = \cos(\theta)$ ,  $f_1 = \sin(\theta)$ :

$$\tan \theta = \sqrt{\frac{\sum_{j=0}^{M_r-1} \lambda_{r_{0j}}^2}{\sum_{j=0}^{M_r-1} \lambda_{r_{1j}}^2}}.$$

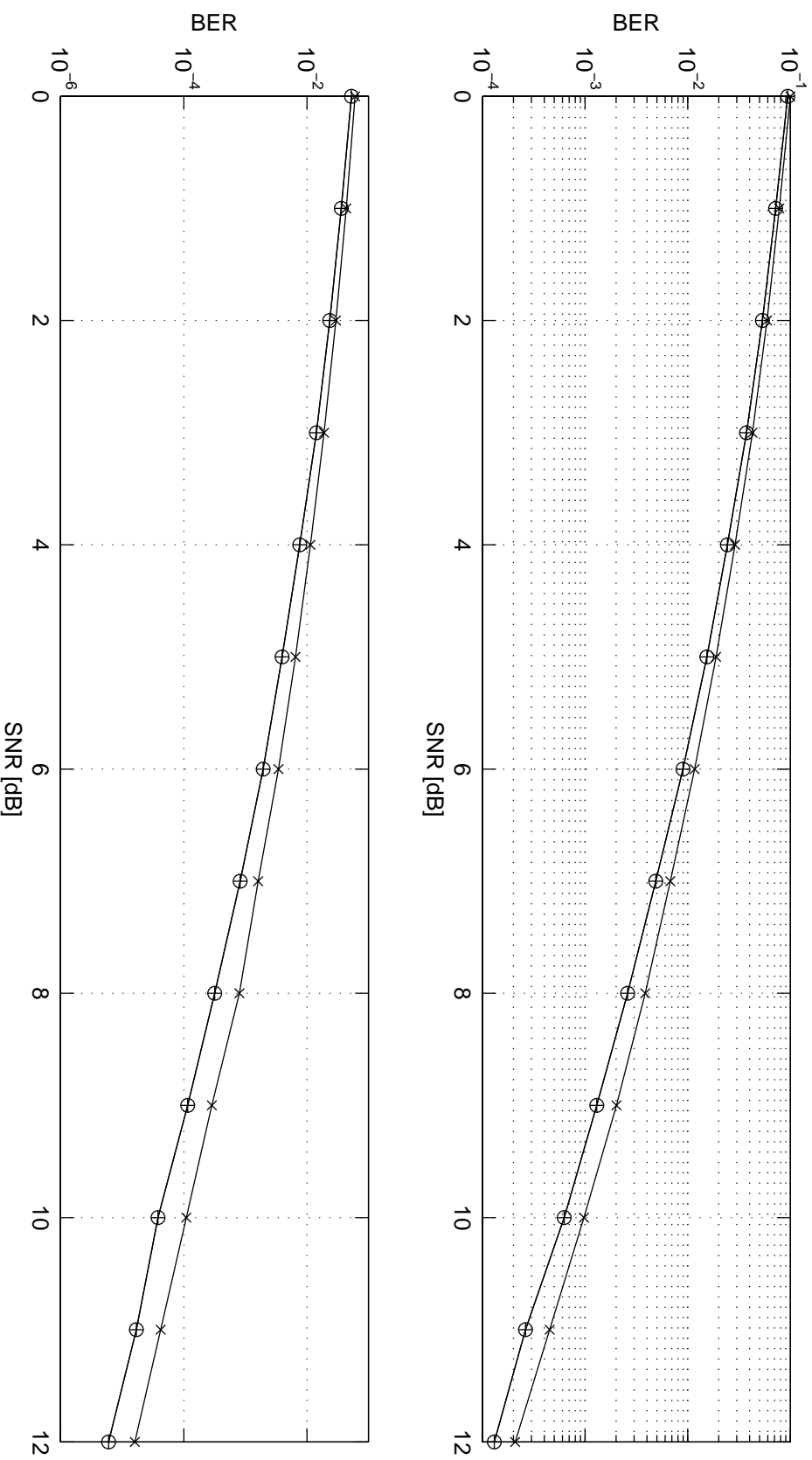
## Simulation: Scenario 1

$M_t = 2$ , Alamouti,  $M_r = 4, 6$ .  $(\mathbf{R})_{k,l} = 0.99^{|k-l|}$  (Non separable)



## Simulation: Scenario 2

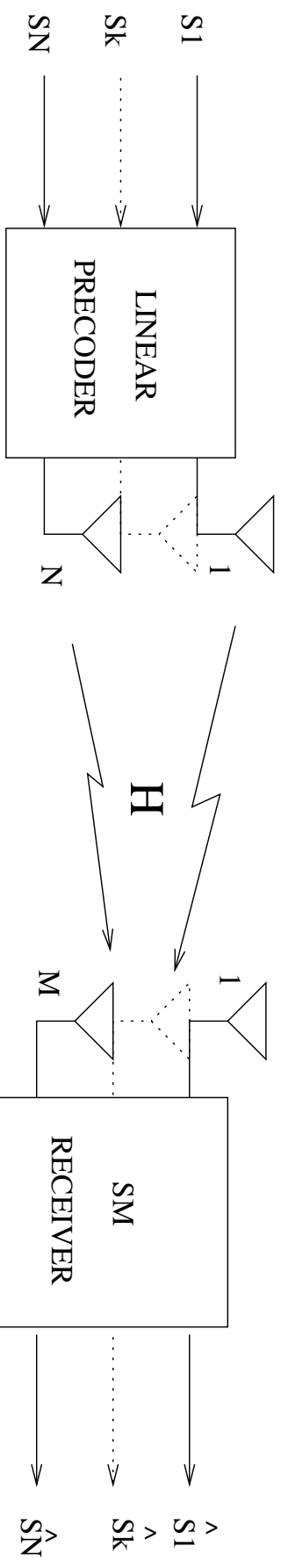
$M_t = 2$ , Alamouti,  $M_r = 4, 6$ . No transmit correlation.  $\mathbf{R}_{r_0} = \mathbf{I}$  and  $\mathbf{R}_{r_1} = \mathbf{I}$  "all-ones".



## Part Two

- **Linear precoding for high-rate codes**

# Spatial Multiplexing with Precoding





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## Proposed approaches

- Focus is on *low-complexity, closed form* solutions for transmitter optimization. (IEEE Globecom 2003, IEEE Trans. Wireless, to appear).
- *Previous work*: Nabar, Bolcskei, Paulraj (BER optimized). Ivrlac et al (capacity optimized).

## Correlated Ricean MIMO model

Channel model:

$$\mathbf{H} = \sqrt{\frac{1}{K+1}} \mathbf{H}_0 \mathbf{R}_\ell^{\frac{1}{2}} + \sqrt{\frac{K}{K+1}} \mathbf{H}_{los}. \quad (13)$$

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad (14)$$

Precoding as a phase/amplitude adjustment:

$$\mathbf{s} = [\sqrt{P_1} s_1 \ \sqrt{P_2} e^{j\phi_2} s_2 \ \dots \ \sqrt{P_{M_t}} e^{j\phi_{M_t}} s_{M_t}]^T. \quad (15)$$

where  $\sum_{i=1}^{M_t} P_i = 1$ .

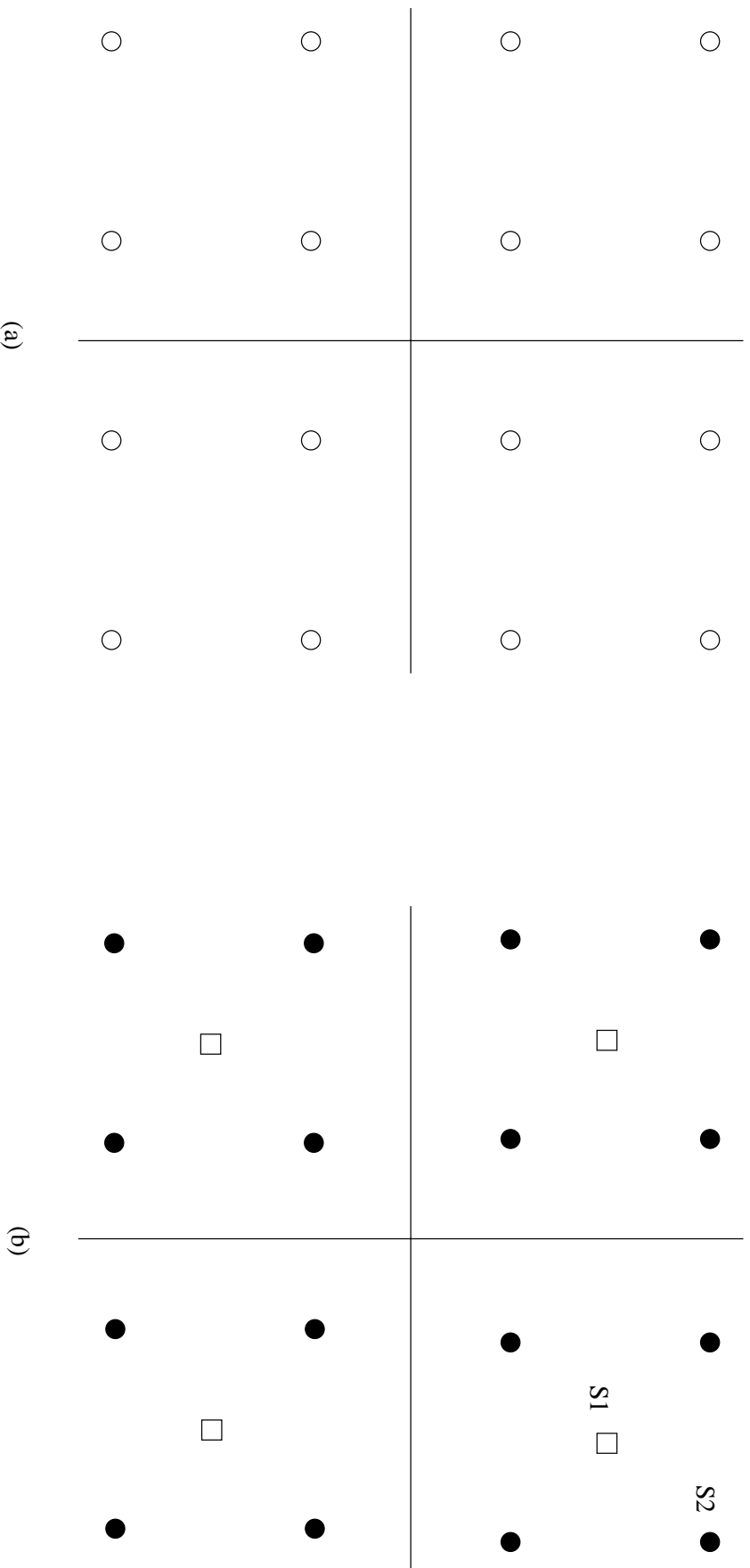
## Constellation multiplexing

let  $s$  belong to a regular  $2^{mM_t}$ -QAM constellation. Then there exist  $r_m \in [0, 1]$  and  $s_1, s_2, \dots, s_{M_t}$  symbols, each belonging to a  $2^m$ -QAM constellation, such that  $s$  can be written in the form of:

$$s = \sum_{k=1}^{M_t} r_m^k s_k \quad (16)$$

where  $r_m$  is such that  $\sum_{k=1}^{M_t} r_m^{2k} = 1$

# Constellation multiplexing



## Precoding for low rank MIMO channels

Consider rank one channel :  $\mathbf{H} = [\mathbf{h}, \mathbf{h}, \dots, \mathbf{h}]$

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{h} \sum_{k=1}^{M_t} \sqrt{P_k} e^{j\phi_k} s_k + \mathbf{n} = \mathbf{h}\mathbf{s} + \mathbf{n} \quad (17)$$

where the precoding coefficients are selected as:

$$P_k = r_m^{2k} \quad k = 1..M_t \quad (18)$$

$$\phi_k = 0 \quad k = 1..M_t \quad (19)$$

⇒ transmitting over a rank one MIMO channel with an appropriate diagonal precoder is equivalent to transmitting over a higher-order constellation signal over a SIMO channel

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## MRC-based SIC detection

### Goals of detection scheme

- Sole purpose of detection scheme is to derive closed-form precoder.
- Precoder should be function only of the long-term statistics  $\mathbf{R}_t$ ,  $K$ ,  $H_{los}$ .
- Ill conditioned channel components dealt with in a MRC-SIC manner rather than matrix inversion.

## E.g.: Closed-form Precoding for 2x2 case

$$\mathbf{H} = \sqrt{\frac{1}{K+1}} \mathbf{H}_0 \begin{bmatrix} \alpha & \beta e^{j\psi} \\ \beta e^{-j\psi} & \alpha \end{bmatrix} + \sqrt{\frac{K}{K+1}} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \quad (20)$$

where  $\alpha^2 + \beta^2 = 1$ , and  $\rho = 2\alpha\beta$  is the modulus of the antenna correlation coefficient

## MRC-SIC detection statistics

$$z_1 = (\mathbf{H}^*)_{1,:} \mathbf{y} = \tau_1 \sqrt{P_1} s_1 + \tau_2 \sqrt{P_2} e^{j\phi_2} s_2 + (\mathbf{H}^*)_{1,:} \mathbf{n} \quad (21)$$

$$\hat{\mathbf{y}} = \mathbf{y} - \mathbf{H}_{:,1} \sqrt{P_1} s_1. \quad (22)$$

$$z_2 = (\mathbf{H}^*)_{2,:} \hat{\mathbf{y}} = \tau_3 \sqrt{P_2} e^{j\phi_2} s_2 + (\mathbf{H}^*)_{2,:} \mathbf{n}, \quad (23)$$

with  $\tau_1 = (\mathbf{H}^*)_{1,:} \mathbf{H}_{:,1}$ ,  $\tau_2 = (\mathbf{H}^*)_{1,:} \mathbf{H}_{:,2}$  and  $\tau_3 = (\mathbf{H}^*)_{2,:} \mathbf{H}_{:,2}$ .



## min distances under average channel behavior

$$\delta_1 = E\{\tau_1\}\sqrt{P_1 d_{min}} - \overline{E\{\tau_2\}}\sqrt{P_2 d_{max}}, \quad (24)$$

$$\delta_2 = E\{\tau_3\}\sqrt{P_2 d_{min}}. \quad (25)$$

where

$$E\{\tau_1\} = \frac{1}{K+1}(2 + K(h_{1,1}^* h_{1,1} + h_{2,1}^* h_{2,1})) \quad (26)$$

$$\overline{E\{\tau_2\}} = \frac{1}{K+1}(2\rho + K|h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}|) \quad (27)$$

$$E\{\tau_3\} = \frac{1}{K+1}(2 + K(h_{1,2}^* h_{1,2} + h_{2,2}^* h_{2,2})). \quad (28)$$

## Precoder optimization via the BBC

"Bit Error Rate Balancing Criterion" (BBC) involves equating minimum distances for symbol detection of  $s_1, s_2$ .

$$P_1 = \frac{[d_{max}(2\rho + K|\alpha|) + d_{min}(2 + K(|h_{1,2}|^2 + |h_{2,2}|^2))]^2}{d_{min}^2(2 + K(|h_{1,1}|^2 + |h_{2,1}|^2)^2 + [d_{max}(2\rho + K|\alpha|) + d_{min}^2(2 + K(|h_{1,1}|^2 + |h_{2,1}|^2))]^2} \quad (29)$$

and

$$P_2 = \frac{d_{min}^2(2 + K(|h_{1,1}|^2 * + |h_{2,1}|^2))}{d_{min}^2(2 + K(|h_{1,1}|^2 + |h_{2,1}|^2)^2 + [d_{max}(2\rho + K|\alpha|) + d_{min}^2(2 + K(|h_{1,1}|^2 + |h_{2,1}|^2))]^2} \quad (30)$$

where  $\alpha = h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}$ .

## Interpretations

No LOS case:

$$P_1 = \frac{(1 + \frac{d_{\max}}{d_{\min}} \rho)^2}{1 + (1 + \frac{d_{\max}}{d_{\min}} \rho)^2}, \quad P_2 = \frac{1}{1 + (1 + \frac{d_{\max}}{d_{\min}} \rho)^2}. \quad (31)$$

- **Uncorrelated:** yields equal power transmission
- **Fully correlated :** eg: 4QAM. Yields  $P_1 = 0.8$  and  $P_2 = 0.2$  (equivalent to 16QAM!)

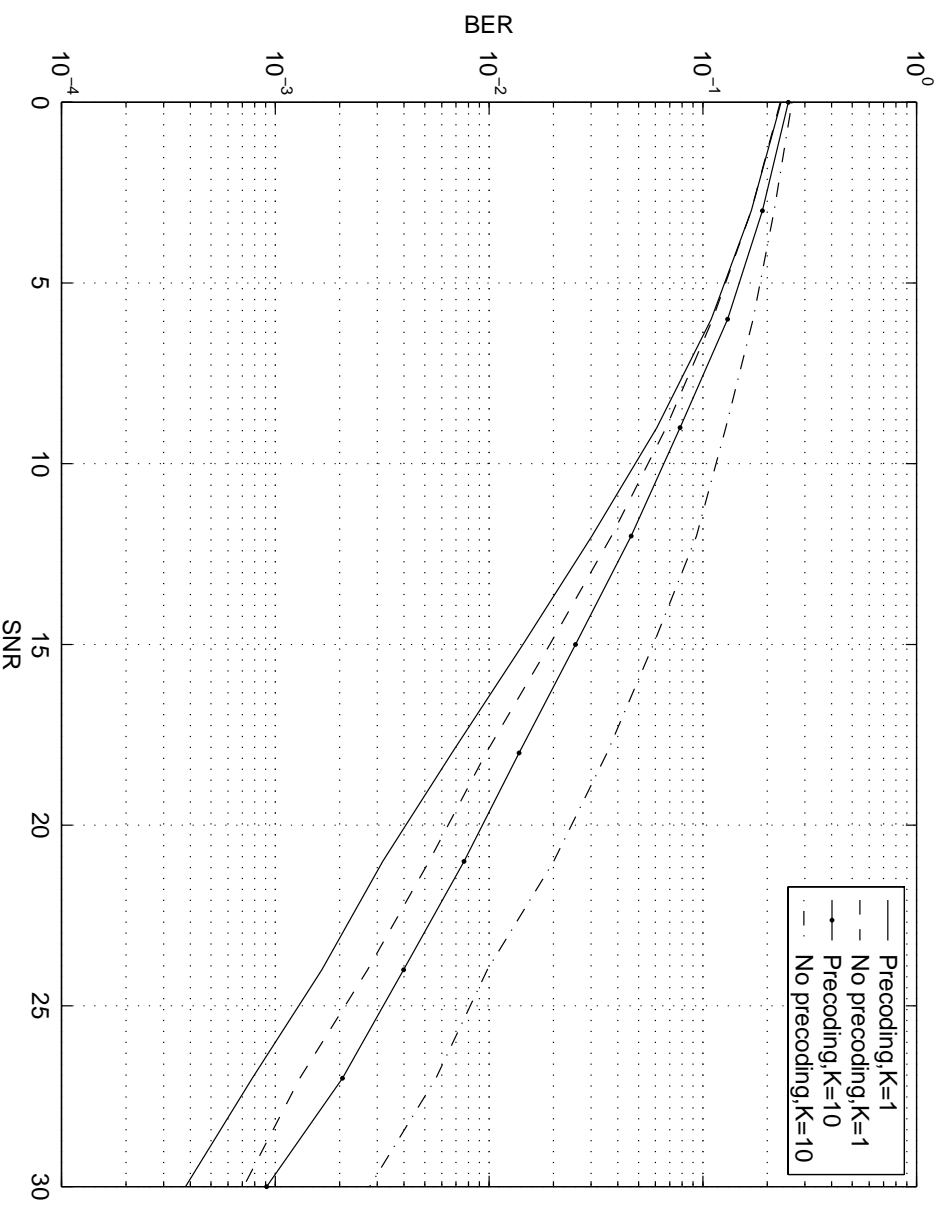
Full LOS case:

- Weights depends on conditioning of LOS part.
- Ill conditioned yields  $P_1 = 0.8$  and  $P_2 = 0.2$ .

**Precoder performs smooth transition between spatial and constellation multiplexing!**

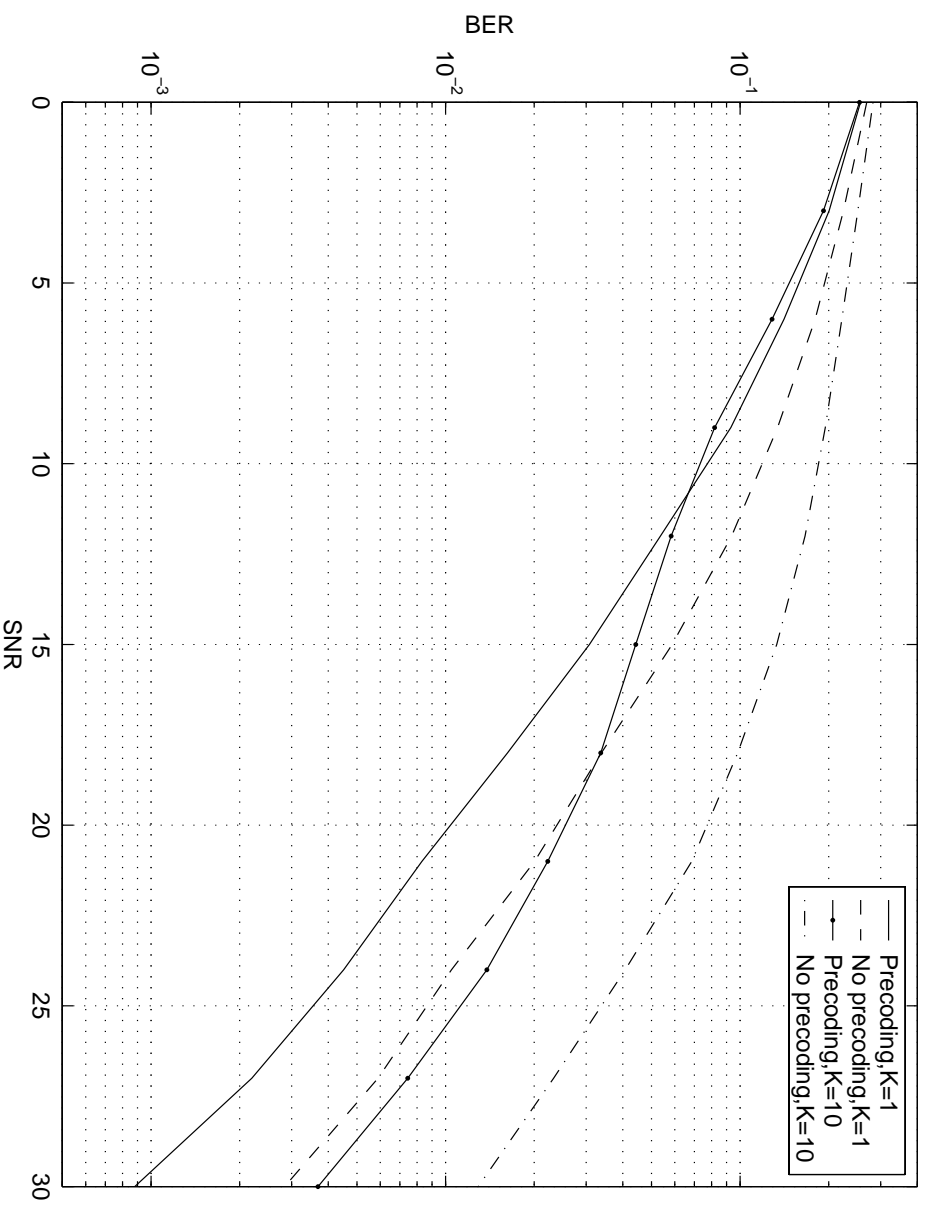
## Numerical results

$K = 1, K = 10$ , MMSE SIC with/without precoding, no TX correlation.



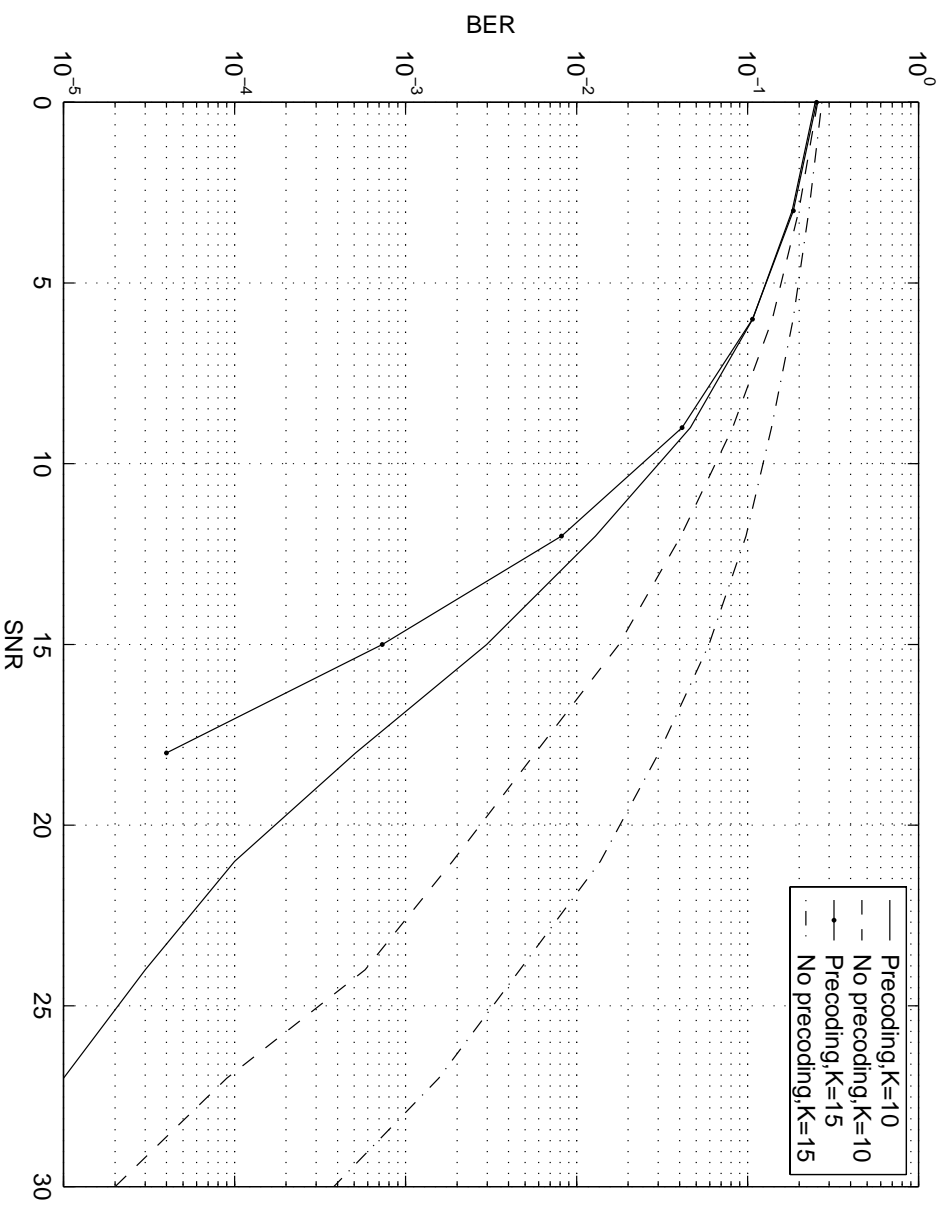
## Numerical results

$K = 1, K = 10$ , MMSE SIC with/without precoding, TX correlation is 0.8.



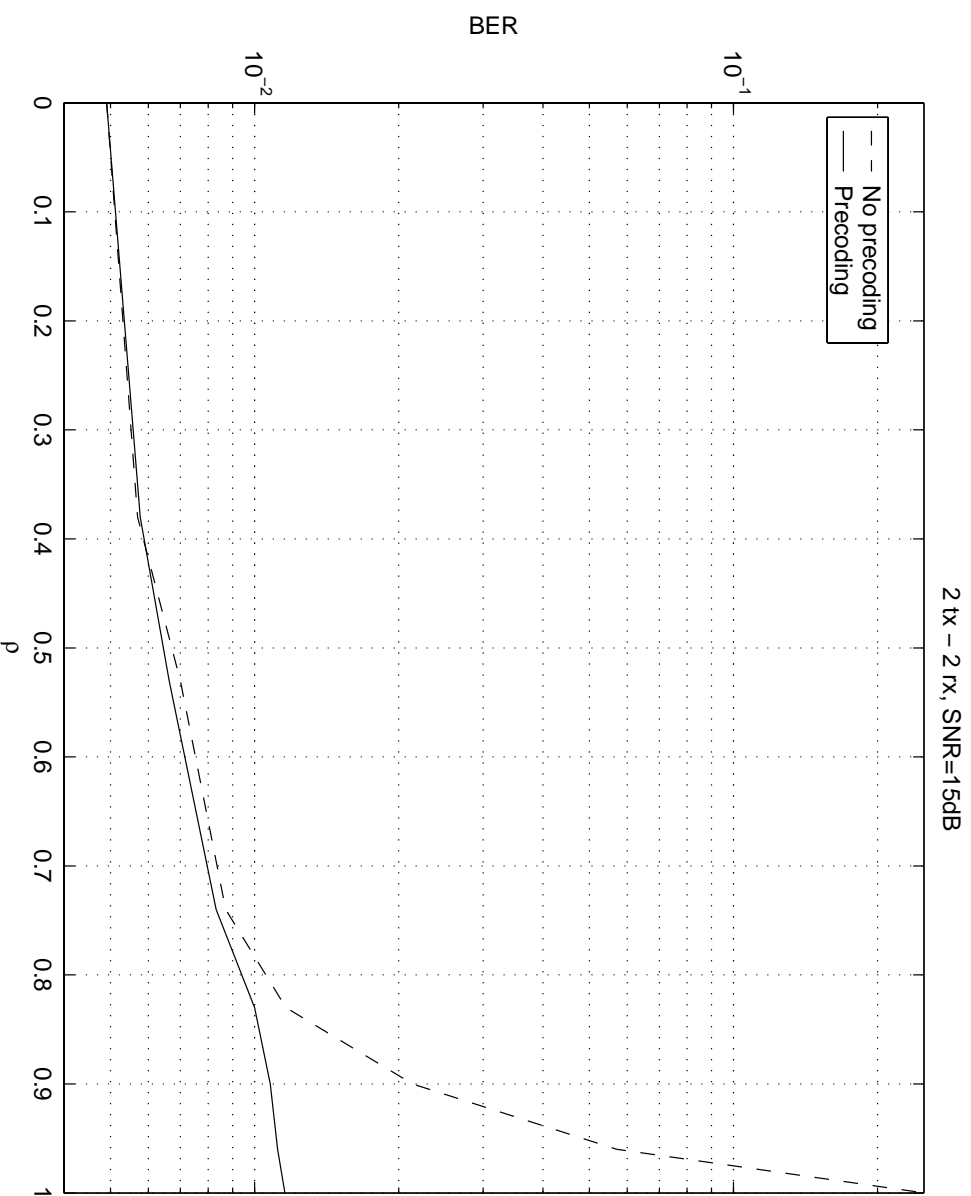
## Numerical results

$K = 10, K = 15$ , ML decoding, no TX correlation.



## Numerical results

$K = 0$ . ML with/without precoding, various TX correlation levels.



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## Conclusions

- Ill-behaved channels are detrimental to MIMO STC performance. A solution is precoding.
- In low-rate STC, gain is limited. Analytical solution difficult, except for macro-diversity case.
- In high-rate STC, gain is large. Analytical (simple) solution is possible. Links spatial muxing to constellation muxing. Can be extended to arbitrary number of antennas (see paper [Akhtar 04] upon request).

Open problems:

- Find closed form precoding for high-rate code, function of RX correlation.
- Find precoder linking STC and SM, function of correlation, Rice (results to appear).