Diversity and Outage Performance of Ricean MIMO Channels

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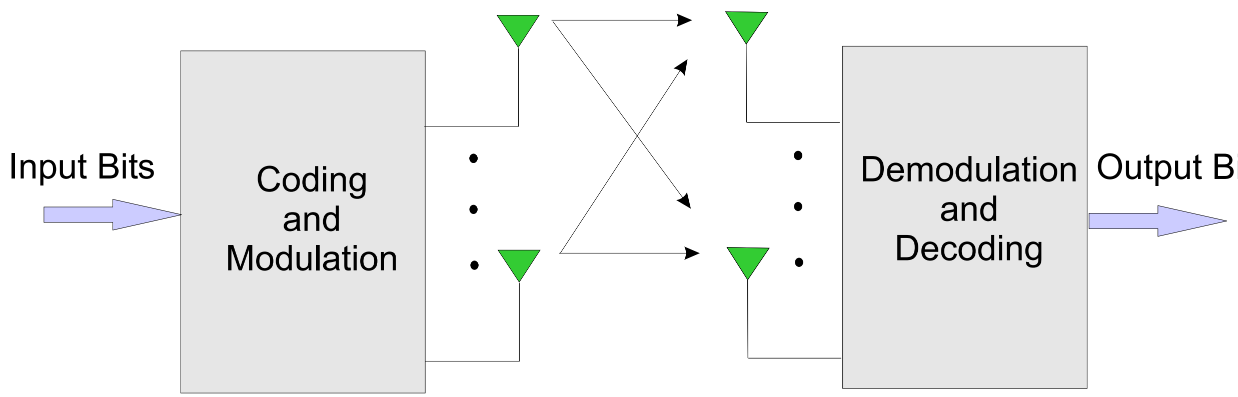
Wireless Multiple-Input Multiple-Output (MIMO) Systems



- **Spatial multiplexing** (Paulraj & Kailath, 1994) a.k.a. **BLAST** (Telatar, 1995, Foschini, 1996) increases **spectral efficiency**
- **Space-time coding** improves **link reliability** through **diversity gain** (Guey et al., 1996, Alamouti, 1998, Tarokh et al., 1998)

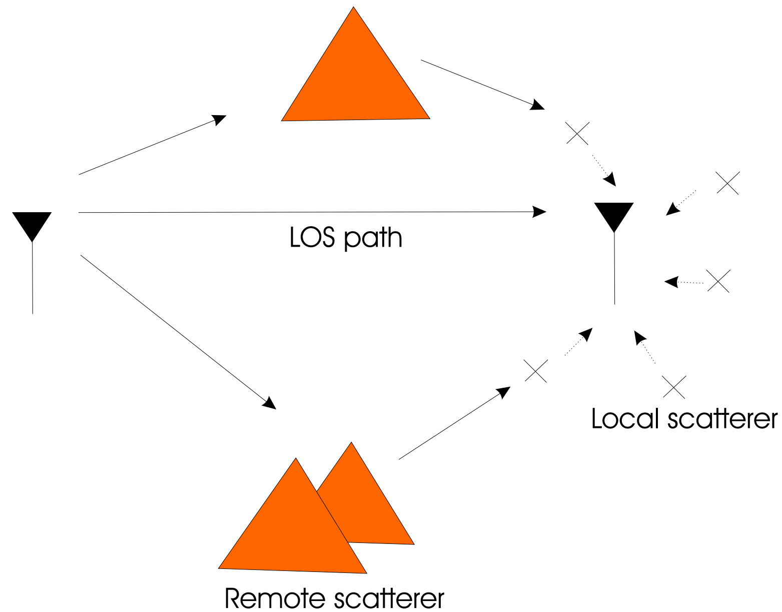
Motivation

- MIMO **performance depends strongly** on **characteristics** of matrix-valued **channel $\mathbf{H}$**
- $\mathbf{H}$ depends on **antenna heights and spacing**, **scattering richness**, **range**, and **antenna polarization**
- MIMO **performance analysis** often assumes **highly idealized** i.i.d. Rayleigh fading channel
- In practice $\mathbf{H}$ may exhibit **Ricean fading** and/or **spatial fading correlation**

$$\Downarrow$$

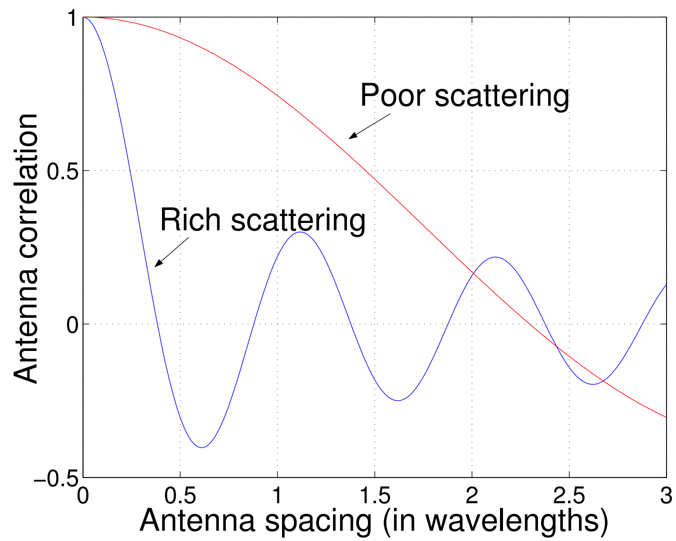
How much diversity gain does a real-world MIMO channel offer?

Ricean Fading



- **Line-of-sight (non-fading)** path between transmitter and receiver
- Ricean K-factor defined as $K = P_{LOS}/P_{SCAT}$
- K depends strongly on **antenna heights** and **range**

Spatial Fading Correlation



- Depends on **richness of scattering** and **antenna spacing**
- Inappropriate antenna spacing or poor scattering results in spatial correlation

MIMO Channel Model

- **Frequency-flat block fading** channel
- $M_R \times M_T$ channel matrix $\mathbf{H}$ is decomposed as

$$\mathbf{H} = \overline{\mathbf{H}} + \widetilde{\mathbf{H}}$$

  - $\overline{\mathbf{H}} = \mathcal{E}\{\mathbf{H}\}$ is **fixed** (possibly line-of-sight) component
  - $\widetilde{\mathbf{H}}$ is **fading** component (circularly symmetric complex Gaussian)
- **Statistics** of $\mathbf{H}$ fully characterized by $\overline{\mathbf{h}} = \text{vec}(\overline{\mathbf{H}})$ and

$$\mathbf{R} = \mathcal{E}\left\{\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}^H\right\} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H \qquad \text{with } \widetilde{\mathbf{h}} = \text{vec}\left(\widetilde{\mathbf{H}}\right)$$

- **Power normalization**

$$\mathcal{E}\{\|\mathbf{H}\|_F^2\} = \|\overline{\mathbf{H}}\|_F^2 + \mathcal{E}\{\|\widetilde{\mathbf{H}}\|_F^2\} = M_T M_R$$

Quantifying Diversity Gain

- $\mathbf{H}$ **perfectly known** at **receiver**, **no channel knowledge** at **transmitter**
- **Utilize all the degrees of freedom** in the channel **to realize diversity gain** (at the expense of spatial rate)
- **Ultimate diversity performance** is characterized by

$$\|\mathbf{H}\|_F^2 = \sum_{i,j} |[\mathbf{H}]_{i,j}|^2$$

- **Effective channel** is **scalar**

$$y = \|\mathbf{H}\|_F^2 \, x + n$$

- This **input-output relation** is obtained with **orthogonal space-time block codes** (or with **MRC** in SIMO channels or **RAKE receiver** in frequency-selective fading SISO channels)

Prior Work

- **MRC-based diversity characterization** for wireless channels (Jakes, 1974, Simon & Alouini, 2000, Wang & Giannakis, 2003, ...)
- **Impact of real-world propagation conditions** on space-time coded transmission (Fitz et al., 1999, Bölcskei & Paulraj, 2000, Uysal & Georghiades, 2001, ...)

Impact of the geometry between fixed and fading channel components on diversity performance unexplored!

Mutual Information

- **Mutual information** given by

$$I = r_s \log_2 \left(1 + \frac{\rho}{M_T} \|\mathbf{H}\|_F^2 \right) \text{ bps/Hz}$$

ρ ... average SNR per receive antenna

r_s ... spatial code rate

- Assume **code word length** is **less than fading block length**

- **Performance** characterized by **distribution of random variable** I

Packet Error Rate and Diversity Order

- **Fixed** transmission **rate** R

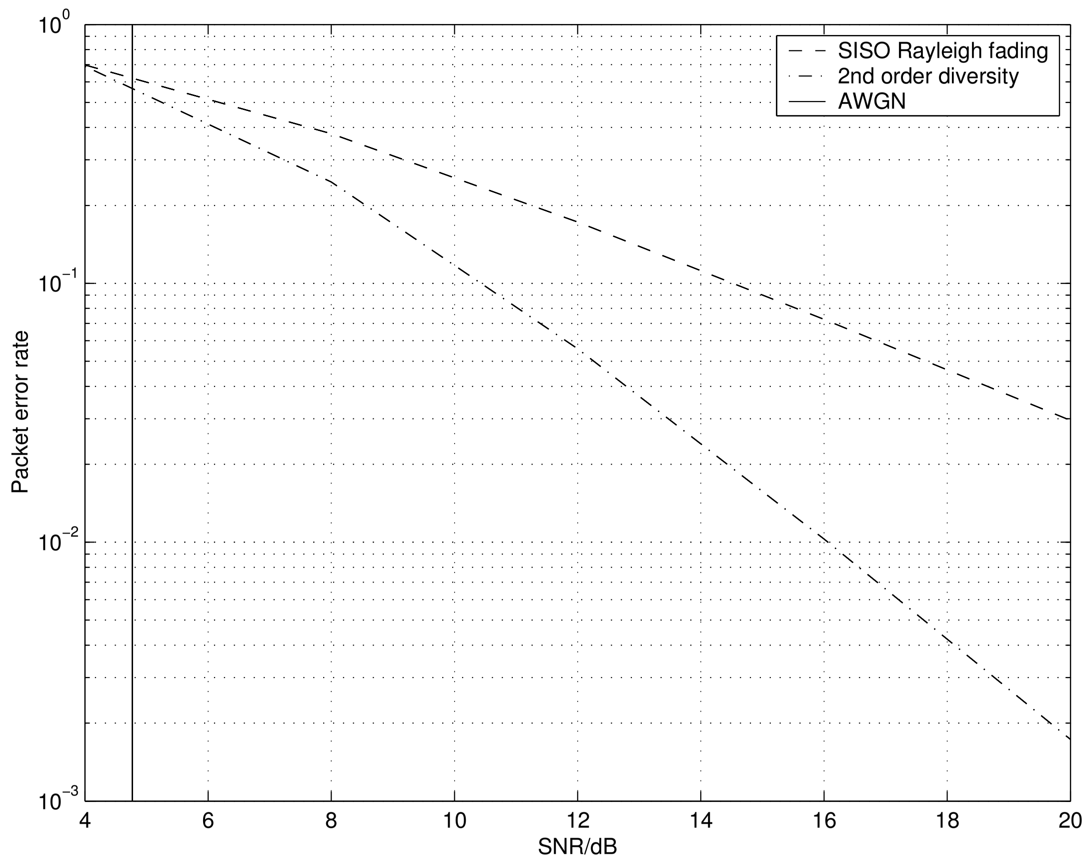
- **Packet error rate (PER)**
  - **packet** is **decoded correctly** if $I \geq R$
  - **packet error** (outage) declared if $I < R$

- Outage probability at rate $R =$ PER at rate R

- **Diversity order** defined as **high-SNR slope** (magnitude) of **PER** (Zheng & Tse, 2003)

$$d(R) = -\lim_{\rho \to \infty} \frac{\log P(I \leq R)}{\log \rho}$$

Packet Error Rate and Diversity Order Cont’d



AWGN channels have diversity order ∞ (Jakes, 1974)

The SISO Channel

- **Single-input single-output** (SISO) channel with

$$h = \overline{h} + \widetilde{h}$$

  and **K-factor**

$$K = \frac{|\overline{h}|^2}{\mathcal{E}\left\{|\widetilde{h}|^2\right\}}$$

- For **arbitrary fixed** transmission **rate** $R > 0$

$$d(R) = \begin{cases} 1, & K < \infty \\ \infty, & K = \infty \end{cases}$$

- No matter how small $\mathcal{E}\left\{|\widetilde{h}|^2\right\}$, there is always a non-zero probability that $\widetilde{h}$ will annihilate $\overline{h}$

Diversity Order in the Ricean MIMO Case: Main Result

For fixed transmission rate R

$$d(R) = \begin{cases} \text{rank}(\mathbf{R}), & \delta = 0 \\ \infty, & \delta > 0 \end{cases}$$

$$\delta \begin{cases} = 0, & \angle(\bar{\mathbf{h}}, \mathcal{R}(\mathbf{R})) = 0 \\ > 0, & 0 < \angle(\bar{\mathbf{h}}, \mathcal{R}(\mathbf{R})) \leq \pi/2 \end{cases}$$

$\mathcal{R}(\mathbf{R})$ ... range-space of $\mathbf{R}$

$\angle(\overline{\mathbf{h}}, \mathcal{R}(\mathbf{R}))$ Defined

- Projection of $\overline{\mathbf{h}}$ onto $\mathcal{R}(\mathbf{R})$

$$\mathbf{p}(\overline{\mathbf{h}}, \mathcal{R}(\mathbf{R})) = \mathbf{R}(\mathbf{R}^H\mathbf{R})^\dagger\mathbf{R}^H\overline{\mathbf{h}}$$

- **Hermitian angle** between $\overline{\mathbf{h}}$ and $\mathcal{R}(\mathbf{R})$ defined as

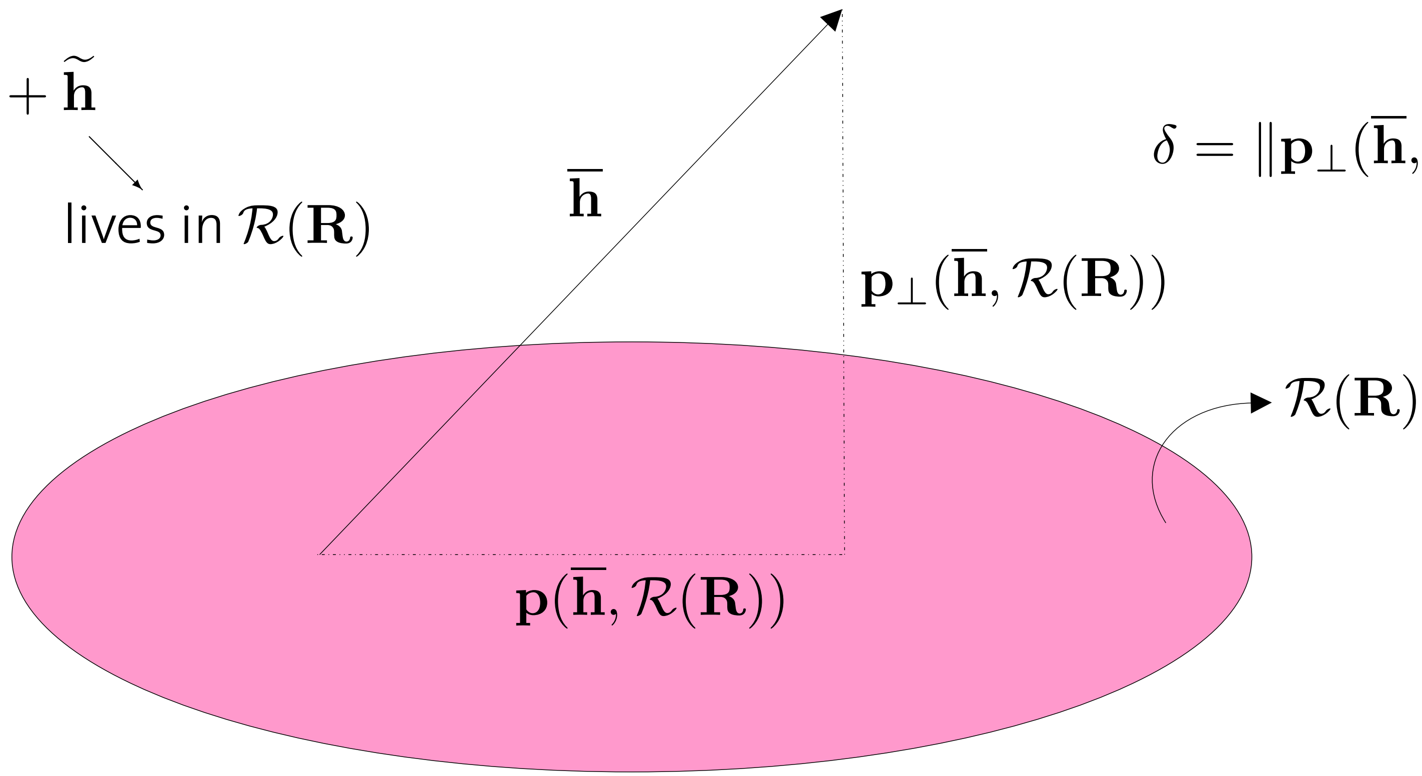
$$\angle(\overline{\mathbf{h}}, \mathcal{R}(\mathbf{R})) = \begin{cases} 0, & \overline{\mathbf{h}} = \mathbf{0}, \\ \cos^{-1}\left(\left|\frac{\overline{\mathbf{h}}^H\mathbf{p}(\overline{\mathbf{h}},\mathbf{R})}{\|\overline{\mathbf{h}}\|\,\|\mathbf{p}(\overline{\mathbf{h}},\mathbf{R})\|}\right|\right), & \overline{\mathbf{h}} \neq \mathbf{0} \text{ and } \mathbf{p}(\overline{\mathbf{h}},\mathbf{R}) \neq \mathbf{0} \\ \pi/2, & \overline{\mathbf{h}} \neq \mathbf{0} \text{ and } \mathbf{p}(\overline{\mathbf{h}},\mathbf{R}) = \mathbf{0} \end{cases}$$

Graphical Interpretation of Main Result

$\mathbf{h} = \overline{\mathbf{h}} + \widetilde{\mathbf{h}}$

lives in $\mathcal{R}(\mathbf{R})$

$\delta = \|\mathbf{p}_\perp(\overline{\mathbf{h}}, \mathcal{R}(\mathbf{R}))\|^2$



If $0 < \angle(\overline{\mathbf{h}}, \mathcal{R}(\mathbf{R})) \leq \pi/\mathbf{2}$, **at least one dimension** of the $M_T M_R$-dimensional vectorized channel **does not experience fading**

Physical Intuition

- **Ricean MIMO channel** is
  - **effectively Rayleigh fading** with rank($\mathbf{R}$) degrees of freedom if $\angle(\overline{\mathbf{h}}, \mathcal{R}(\mathbf{R})) = 0$
  - **effectively AWGN** if $\angle(\overline{\mathbf{h}}, \mathcal{R}(\mathbf{R})) \neq 0$
- Can we **signal with zero outage** over **Ricean MIMO channels** if $\angle(\overline{\mathbf{h}}, \mathcal{R}(\mathbf{R})) \neq 0$?

Critical Transmission Rate

- **Packet error rate** (outage probability) at transmission rate R satisfies

$$P(I \leq R) \begin{cases} = 0, & R < R_{crit} \\ > 0, & R \geq R_{crit} \end{cases}$$

  with

$$R_{crit} = r_s \log_2\left(1 + \frac{\delta}{M_T}\rho\right)$$

- For $R \leq R_{crit}$, **Ricean MIMO channel** behaves **like an AWGN channel** (zero outage)

- R_{crit} is the **rate supported by** the **non-fading component** of the channel

Spatial Fading Correlation – Good or Evil ?

- **Always detrimental in the case of pure Rayleigh fading**
  - Mild correlation (i.e., rank$(\mathbf{R}) = N$) results in **coding gain loss** (i.e., **offset** in PER vs. SNR curve)
  - Extreme correlation (i.e., rank$(\mathbf{R}) < N$) results in **reduction of diversity order**
- **Answer depends critically on $\angle(\overline{\mathbf{h}}, \mathbf{R})$ in the case of Ricean fading**
  - Mild correlation (i.e., rank$(\mathbf{R}) = N$) results in **coding gain loss** (relative geometry between $\overline{\mathbf{h}}$ and $\mathbf{R}$ critical in determining loss!)
  - Extreme correlation (i.e., rank$(\mathbf{R}) < N$) can be **highly beneficial**

Numerical Example – Setup

- $M_T = 2, M_R = 1$
- Channel 1

$$\overline{\mathbf{H}} = \sqrt{\frac{K}{1+K}} \begin{bmatrix} 1 & 1 \end{bmatrix}, \qquad \widetilde{\mathbf{H}} = \sqrt{\frac{1}{1+K}} \begin{bmatrix} \widetilde{g}_{1,1} & \widetilde{g}_{1,2} \end{bmatrix}$$

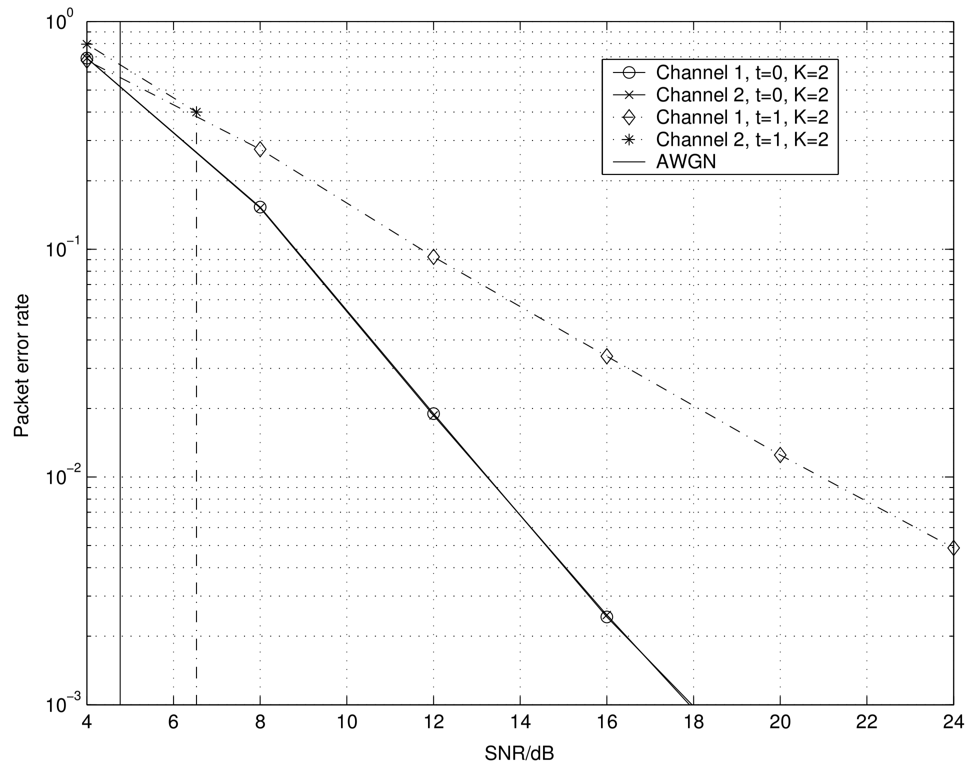
- Channel 2

$$\overline{\mathbf{H}} = \sqrt{\frac{K}{1+K}} \begin{bmatrix} 1 & -1 \end{bmatrix}, \qquad \widetilde{\mathbf{H}} = \sqrt{\frac{1}{1+K}} \begin{bmatrix} \widetilde{g}_{1,1} & \widetilde{g}_{1,2} \end{bmatrix}$$

- Transmit correlation

$$t = \mathcal{E}\{\widetilde{g}_{1,1}\, \widetilde{g}_{1,2}^*\}$$

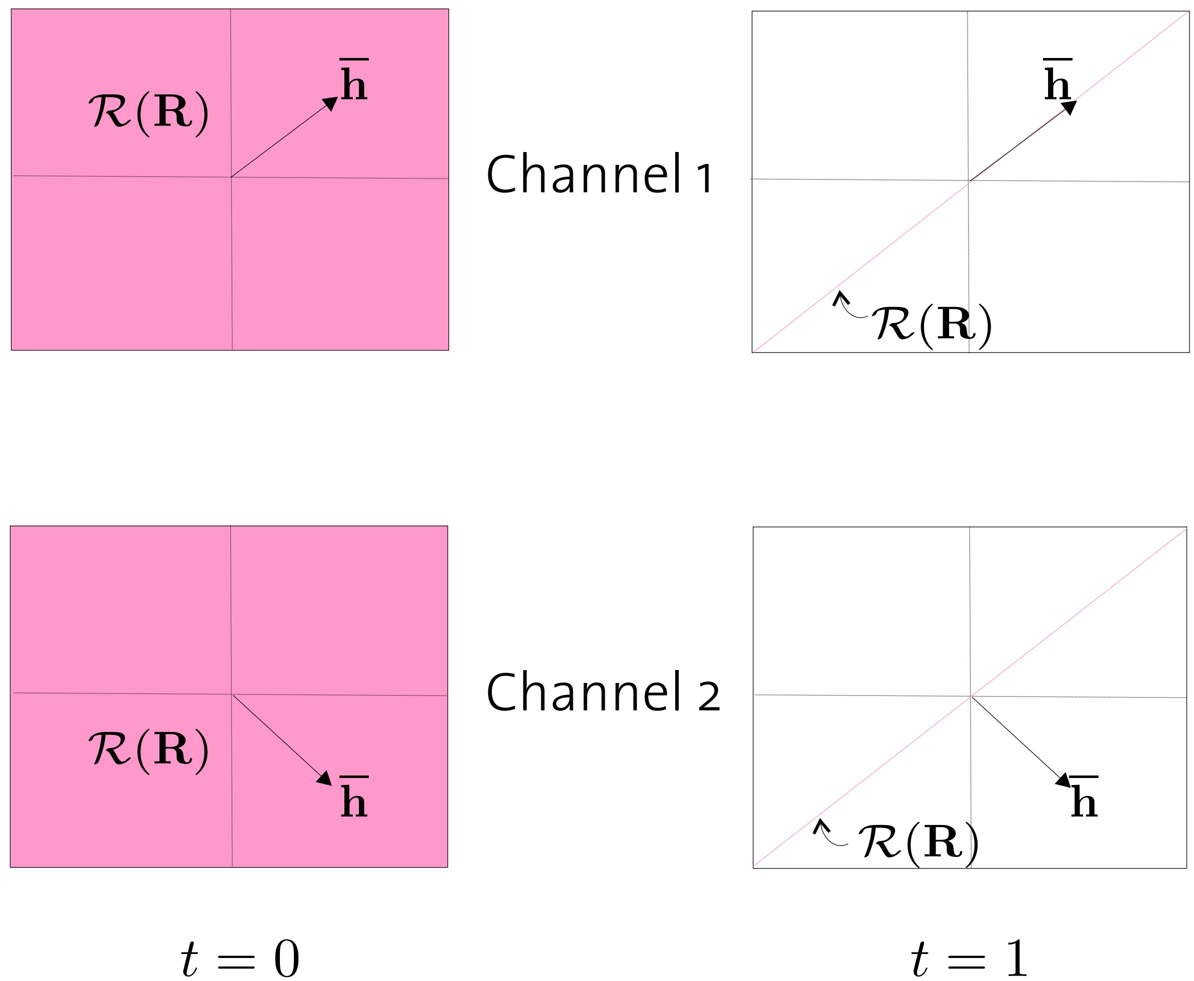
Numerical Example



$$R = 2 \text{ bps/Hz}$$

In the correlated case Channel 2 behaves like an AWGN channel

Numerical Example: Interpretation



Diversity Order Through Symbol Error Rate

- **Diversity order** is often defined as **high-SNR slope** (magnitude) of **uncoded symbol error rate**

- This definition yields

$$d_E = \begin{cases} \text{rank}(\mathbf{R}), & \delta = 0 \\ \infty, & \delta > 0 \end{cases}$$

(same δ as before)

- Exhibits **same behavior** as **diversity order** defined through **outage analysis**

Comments on Wideband Spectral Efficiency

- **Wideband spectral efficiency characterized by** (Verdu, 2002)
  - $(E_b/N_o)_{min}$ ... **minimum SNR-per-bit** required to sustain error free communication
  - $\mathcal{S}$ ... **slope** of spectral efficiency (in bps/Hz/3dB) at $(E_b/N_o)_{min}$
- For orthogonal space-time block codes in the wideband regime

$$(E_b/N_o)_{min} = \ln 2/(r_s M_R)$$

$$\mathcal{S} = 2r_s \left(\left(\frac{\text{Tr}(\mathbf{R}^2) + 2\overline{\mathbf{h}}^H \mathbf{R}\overline{\mathbf{h}}}{(M_T M_R)^2} \right) + 1 \right)^{-1}$$

- $\mathcal{S}$ impacted by the "**peakiness**" of the fading distribution which is turn depends on relative geometry between $\overline{\mathbf{h}}$ and $\mathbf{R}$

Conclusions

- **Ultimate diversity performance** in **general MIMO channels** characterized by

$$d = \begin{cases} \text{rank}(\mathbf{R}), & \delta = 0 \\ \infty, & \delta > 0 \end{cases}$$

- Established the presence of a **critical transmission rate** R_{crit} **below which** the **Ricean MIMO channel behaves like an AWGN channel**

- R_{crit} is **zero** for **SISO channels** and for **purely Rayleigh fading MIMO channels**

- R_{crit} depends on $\angle(\overline{\mathbf{h}}, \mathcal{R}(\mathbf{R}))$

- The **notion of diversity order** should be **"handled with care"**