Capacity Enhancements Using Space-Time Collaborative Communications in Wireless Channels

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Outline

- Introduction
- Distributed Communications
- The Model
- Main Results
- Conclusions

Introduction

- The challenge for wireless data is that the physical limitations of wireless channels makes it hard to provide:
  - High rates at large distances,
  - Low cost devices, and
  - Mobile applications at low costs.
- These limitations include
  - Path loss,
  - Spectrum limitations
  - Power limitations
  - interference
  - fading, etc.

Smart Antennas/MIMO

- In addressing these problems many signal processing techiques have been introduced.
- In particular, antenna technologies have attracted some attention.
- These techniques require multiple antennas at the transmitter and/or the receiver.
- Smart antenna techniques include
  - Beamforming/SDMA, and
  - Space-time coding
- These techniques have been/are being extensively studied.
- They are known to be capable of producing significant gains.

Distributed Communications

- However in some applications it may be hard to deploy multiple transmit/receive antenna on wireless devices.
- For instance ad hoc sensor elements may not be easily endowed with multiple antennas due to their small size.
- For these applications, it is interesting to see if the benefits of multiple antennas can be obtained from single antenna devices in a distributed way.
- Distributed communications may have other applications such as that of design of more intelligent relay elements.
- It can also have a lot of impact on the higher layers of communications.

Distributed Beamforming

- In this paradigm, ad hoc antenna arrays collaborate to produce a directional beam pattern or to produce effects analogous to those of SDMA.
- Distributed beamforming has been implicitly studied in the literature in 60's in the context of random arrays by amongst others, S.D. Steinberg and Y.T. Lo.
- Their research goal was to show that random geometry arrays are more efficient than uniform arrays.
- However, their results also apply to distributed arrays, but
  - distributed arrays are much harder to implement.

Distributed STC

- In contrast, distributed space-time coding has not been extensively studied before.
- To the best of my knowledge, the initial work on collaborative communications stretches as far back as the pioneering papers by van der Mullen (1971) and Cover et al. (1979) on the relay channel.
- However, the results obtained there do not appear to be directly applicable to wireless networks.
- This is because in realistic wireless models, it is impossible to transmit and receive on the same antenna on the same band simultaneously (half-duplex constraint).

Distributed STC

- Furthermore, the channel is usually not known to the transmitting nodes; only the receiving nodes have knowledge of the channel.
- Further extensions of the relay channel has been done by Schein (2000),Gupta (2003), Gastpar (2002), Sendonaris (2003) et al.
- Some more recent work on collaborative communications with emphasis on treating the wireless channel was done by Laneman (2003), Hunter (2003), Khojastepour (2003) et al.

Distributed STC: Previous Work

- Laneman et al. consider a two stage communications approach (where the source transmits for a fixed amount of time followed by a fixed length relaying phase) to solve the half-duplex constraint and consider repetition and space-time based cooperative diversity algorithms. They extend their work with the consideration of adaptive protocols such as selection relaying and incremental relaying.
- Hunter (2003) et al. consider a similar TDMA approach where the relay is permitted to transmit its own information during the second phase if it is unable to collaborate.

Distributed STC: Previous Work

- Khojastepour et al. assume two dedicated orthogonal subchannels between two mobile users and derive an achievable region for communication to a base station. More recent work considers a protocol that is more bandwidth efficient based on the relay joining the transmission after a *fixed* amount of time.

The Present Work

- A bandwidth efficient approach: that does not employ predetermined TDMA or orthogonal subchannels to resolve the half-duplex constraint; each relay determines based on its own receive channel when to listen and when to transmit.
- The transmitters are not aware of the channel and we make no assumption of degradedness; the noise at the relays is independent of that at the destination.
- Asynchronousity is allowed: our results still hold under a bounded asynchronous model.
- Relay assisting each other: our approach permits one relay to assist another in receiving the message, a feature not present in previous work.

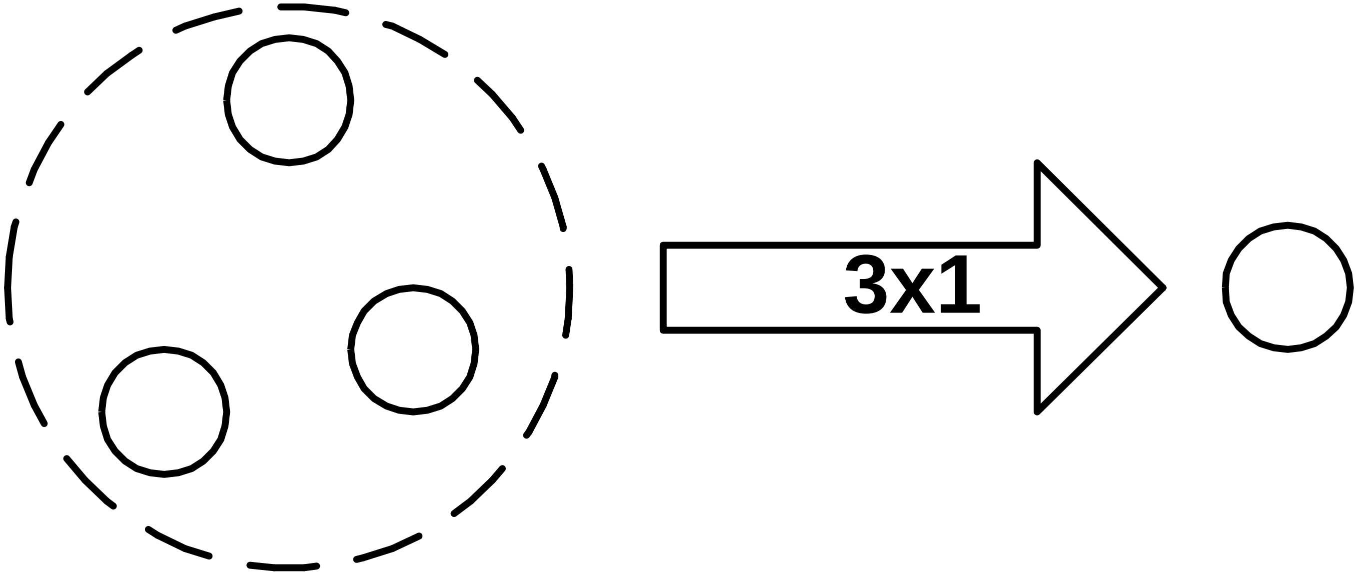
The Present Work

- Our primary interest in this paper is to determine if we can achieve the genie bound on diversity, the diversity gain that would be achieved if all the transmit antennas of the source and relay nodes were in fact connected to a single node.
- Laneman et. al refer to this limit the transmit diversity bound.
- For example, suppose we consider the 3 transmit collaborators and 1 receiver node scenario. If all the collaborators were aware of the message *a priori*, we could in principle achieve the ideal performance of a 3x1 space-time system between the transmit cluster and the receiver node.
- However, only the source node in the transmit cluster is aware of the message *a priori*. The other two nodes in the cluster must serve as relays and are not aware of the message *a priori*.

There will be a loss in performance (as measured by the probability of outage) compared to the idealized 3x1 space-time system.

- In particular, we shall be interested in determining sufficient conditions on the geometry and signal path loss of the transmitting cluster for which performance close to the genie bound can be guaranteed.

3 Collab. Transmitters



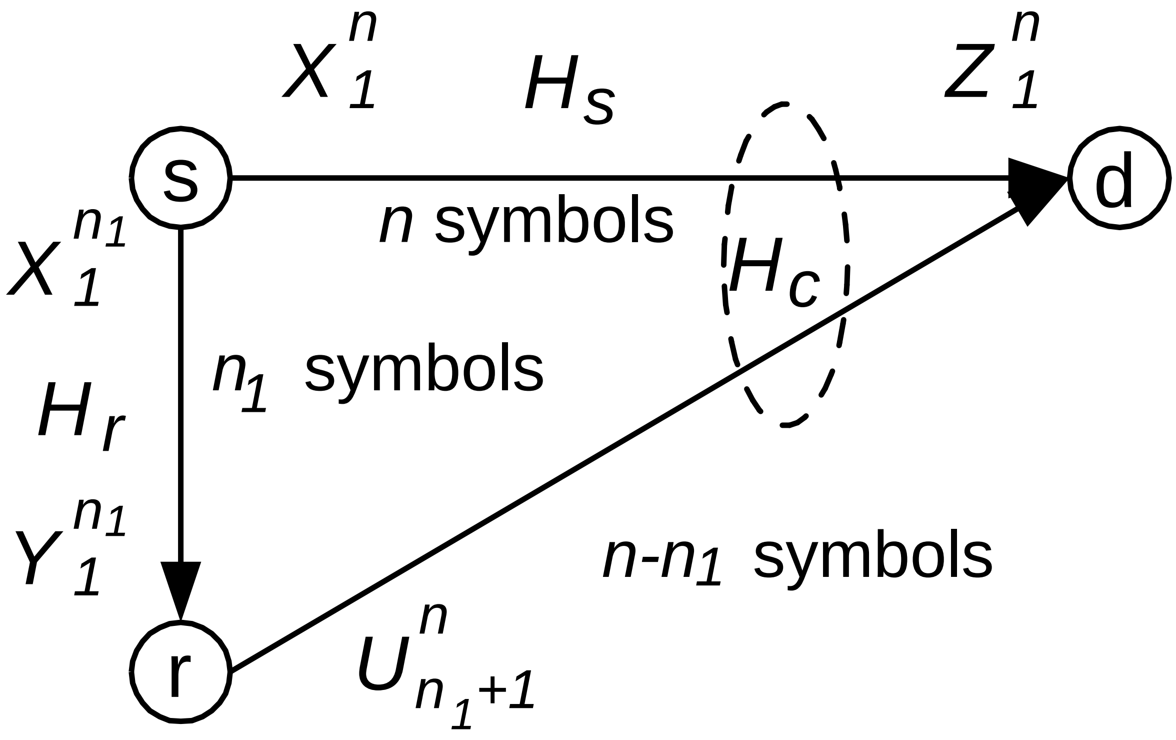
Our Model

- For simplicity, we consider 3 nodes denoted as source (s), relay (r) and destination (d) each equipped with N_s, N_r and N_d antennas respectively (the results readily generalize to multiple relay/receive nodes). We assume that while listening to the channel, the relay may not transmit. Hence, the communications protocol we propose is as follows.
- The source node wishes to transmit one of 2^{nR} messages to the destination employing n channel uses. While transmitting, the relay node listens. Due to the relay node's proximity to the source, after n_1 samples from the channel (a number which the relay determines on its own and for which the source has no knowledge), it may correctly decode the message.

Our Model

- After decoding the message, it then proceeds to transmit for the remaining $n - n_1$ transmissions in an effort to improve the reception of the message at the destination. The destination is assumed to be made aware of n_1 before attempting to decode the message.
- This may be achieved for instance, by an explicit low-rate transmission from the relay to the destination.

The Model


X_1^n
H_s
Z_1^n
s
d
n symbols
$X_1^{n_1}$
H_c
n_1 symbols
H_r
$Y_1^{n_1}$
n-n_1 symbols
r
$U_{n_1+1}^n$

The Channel Model

- We assume that all channels are modelled as AWGN with quasi-static fading. In particular, if X and U are the transmission from the source and relay nodes respectively and we denote by Y and Z the received messages at the relay and destination respectively, then during the listening phase we have that

$$\begin{aligned} Z &= H_s X + N_Z \\ Y &= H_r X + N_Y \end{aligned}$$

where the N_Z and N_Y are independent column vectors of complex additive white Gaussian noise with variance 1/2 per row per dimension.

- During the collaboration phase, we have that

  $$Z = H_c[X, U]^T + N_Z$$

  where H_s is a submatrix of H_c.

The Channel Model

- We further assume that the source has no knowledge of the H_r and H_c matrices (and hence the H_s matrix too). Similarly, the relay has no knowledge of H_c but is assumed to know H_r. Finally, the destination knows H_c.
- It will be assumed that all transmit antennas have unit average power during their respective transmission phases. The receive antennas noises are unit power Gaussian. If this is not the case, the respective H matrices may be appropriately scaled row-wise and column-wise.
- Under the above assumptions, it is well known that a MIMO system with rate R bits/channel use can reliably communicate over any channel with transfer matrix H such that $R < \log_2 \det(I + HH^\dagger) \overset{\triangle}{=} C(H)$.

- We also define $C(H, \gamma) \overset{\triangle}{=} \log_2 \det(I + \gamma H H^\dagger)$

The Channel Model

- Intuition for the above problem then suggests the following. During the listening phase, the relay knowing H_r listens for an amount of time n_1 such that $nR < n_1 C(H_r)$. During this time, the relay receives at least nR bits of information and may reliably decode the message.
- The destination, on the other hand, receives information at the rate of $C(H_s)$ bits/channel use during the listening phase and at the rate of $C(H_c)$ bits/channel use during the collaborative phase. It may reliably decode the message provided that $nR < n_1 C(H_s) + (n - n_1) C(H_c)$.

The Channel Model

- In the limit as $n \to \infty$, the ratio n_1/n approaches a fraction f and we may conjecture that there exists a "good" code of rate R for the set of channels (H_r, H_c) which satisfy

$$\begin{aligned} R &\leq fC(H_s) + (1-f)C(H_c) \\ R &\leq fC(H_r) \end{aligned}$$

We note that if the channel between the source and the relay is particularly poor, we may fall back on the traditional point-to-point communications paradigm and add the following region to the above.

$$R \leq C(H_s)$$

Discussion

- The above intuition is not a proof of the achievability but it does provide an upper bound on the performance of any code for which we require the relay to either correctly decode the message or to remain silent.
- The essential difficulty in proving that there exists a code which is "good" for any such pair of channels (H_r, H_c) is two-fold. The problem we are dealing with is a relay channel which is also a compound channel: we seek to prove the existence of one code which performs well over an entire set of channels (unknown to the transmitters).

Discussion

- The key will be to show the existence of a code that may essentially be refined. Regardless of the actual value of n_1, there exists a codebook for the source which, starting at time $n_1 + 1$, may be layered with the transmission of the relay and perform just as well as if the value of n_1 had been known to the source.
- To address these problems, we had to invent some new information theoretic methods.
- These are too technical to be discussed here.
- We summarize our results in the next slides.

Result I

- (Synchronous collaborative communications): Let $||H|| = \max_{i,j}\{|H_{i,j}|\}$. Consider the set $\mathcal{H}_{\delta,L}(R)$ of matrices (H_r, H_c) such that $||H_r|| \leq L$ and $||H_c|| \leq L$ and which satisfy either

$$\begin{aligned} R+\delta &\leq fC(H_s)+(1-f)C(H_c) && (1) \\ R+\delta &\leq fC(H_r) && (2) \end{aligned}$$

or

$$R+\delta \leq C(H_s) \tag{3}$$

for some $\delta \leq f \leq 1-\delta$ (each f may depend on the pair (H_r, H_c)). Then, the rate R is achievable for the compound relay channel $\mathcal{H}_{\delta,L}(R)$ for any $\delta > 0$ and $L > 0$.

- The above theorem essentially gives the intuitive region.

Result II

- In the case that the transmissions from the relays arrive asynchronously at the receiver but this asynchronousity is bounded by a known value D, then we have proved that
- (Asynchronous collaborative communications): The same rates are achievable as in the synchronous case.

Upperbound on Diversity Order

- Consider a total of K collaborators, each equipped with N_i antennas, $1 \leq i \leq K$ and a receiver node equipped with N_r antennas.
- We upperbound the performance of any scheme by supposing that a genie has revealed to each collaborator the actual message W that is to be transmitted. While each collaborator will have a different path loss based on node geometry, we may further upperbound the performance by supposing that all nodes have the best path loss.
- In such a case, since all paths between collaborators/receiving node are fading, it is clear that no collaborative scheme performs better than a $N_t \times N_r$ MIMO system in terms of diversity order where $N_t = \sum_{i=1}^{K} N_i$.

Upperbound on Diversity Order

- (Theorem III) In a collaborative scheme with K collaborators, each equipped with N_i antennas and a receiver node with N_r antennas, the diversity order is upperbound by $N_t \times N_r$ where $N_t = \sum_i N_i$.

Higher Number of Collaborators

- The results in Theorems 1 and 2 generalize in a straight forward manner to multiple transmit collaborators. In particular, we now consider 3 transmit collaborators and 1 receiver, all equipped with a single antenna.
- There, the channel between two nodes, say nodes s and r_1, will be denoted by H_{s,r_1}. Likewise, the channel between the pair of nodes (s, r_0) and the node r_1 will be denoted H_{sr_0,r_1} with H_{s,r_1} a submatrix of H_{sr_0,r_1}.
- With this notation in hand, communication is successful provided that either

Higher Number of Collaborators

- $$R \leq f_0 C(H_{s,r_0}, G\gamma)$$

  $$R \leq f_0 C(H_{s,r_1}, G\gamma) + f_1 C(H_{sr_0,r_1}, G\gamma)$$

  $$R \leq f_0 C(H_{s,d}, \gamma) + f_1 C(H_{sr_0,d}, \gamma) + (1 - f_0 - f_1) C(H_{sr_0 r_1,d}, \gamma)$$

  for some $f_0 > 0$, $f_1 > 0$ with $f_0 + f_1 < 1$ or

  $$R \leq f_0 C(H_{s,r_0}, G\gamma)$$

  $$R \leq f_0 C(H_{s,d}, \gamma) + (1 - f_0) C(H_{sr_0,d}, \gamma)$$

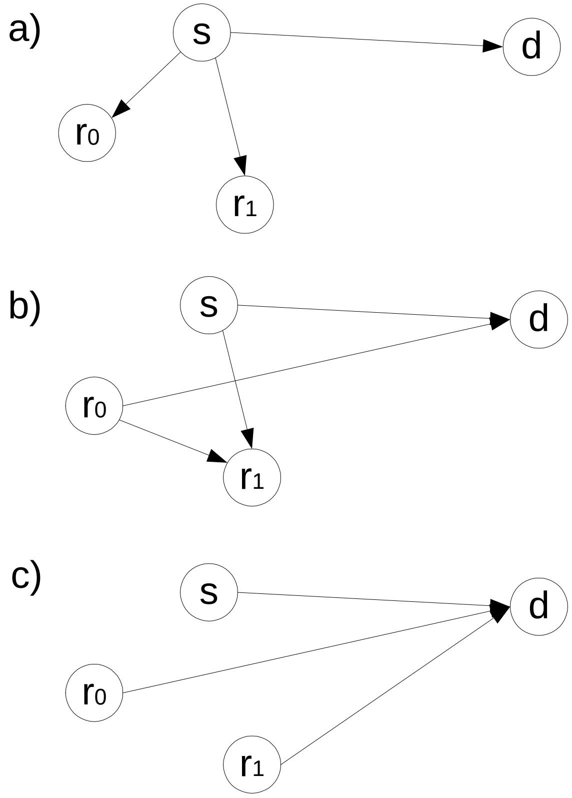
  for some $0 < f_0 < 1$ or

  $$R < C(H_{s,d}, \gamma)$$

Higher Number of Collaborators

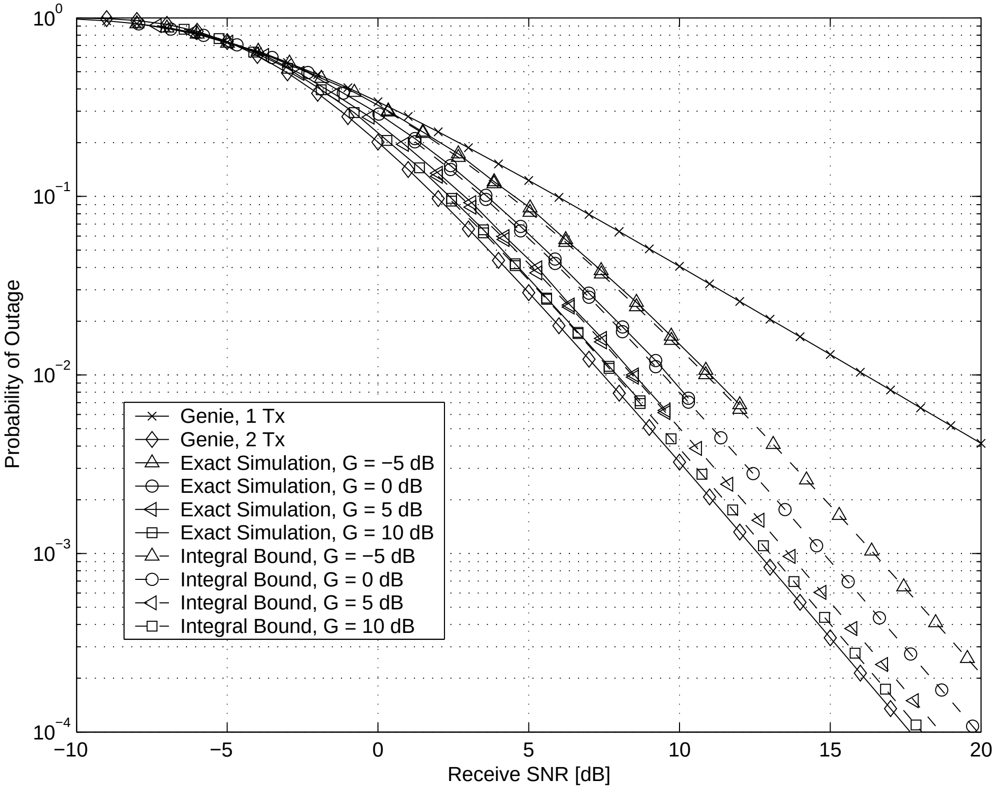
- In the above, we have assumed without loss of generality that $C(H_{s,r_0}, G\gamma) > C(H_{s,r_1}, G\gamma)$. If this is not the case, then we must add two more regions to the achievable compound channel (which may be obtained by symmetrically interchanging r_0 and r_1 in the above regions).
- Furthermore, f_0 is greedily chosen by node r_0 to be the smallest value which satisfies the above equation. Likewise, f_1 is then chosen as the smallest value which satisfies the above equation.
- If the number of collaborative nodes were further increased to m, we would see a cascade effect by which the relays would quickly share among themselves the message by way of $m(m-1)/2$ possible paths.

The Model



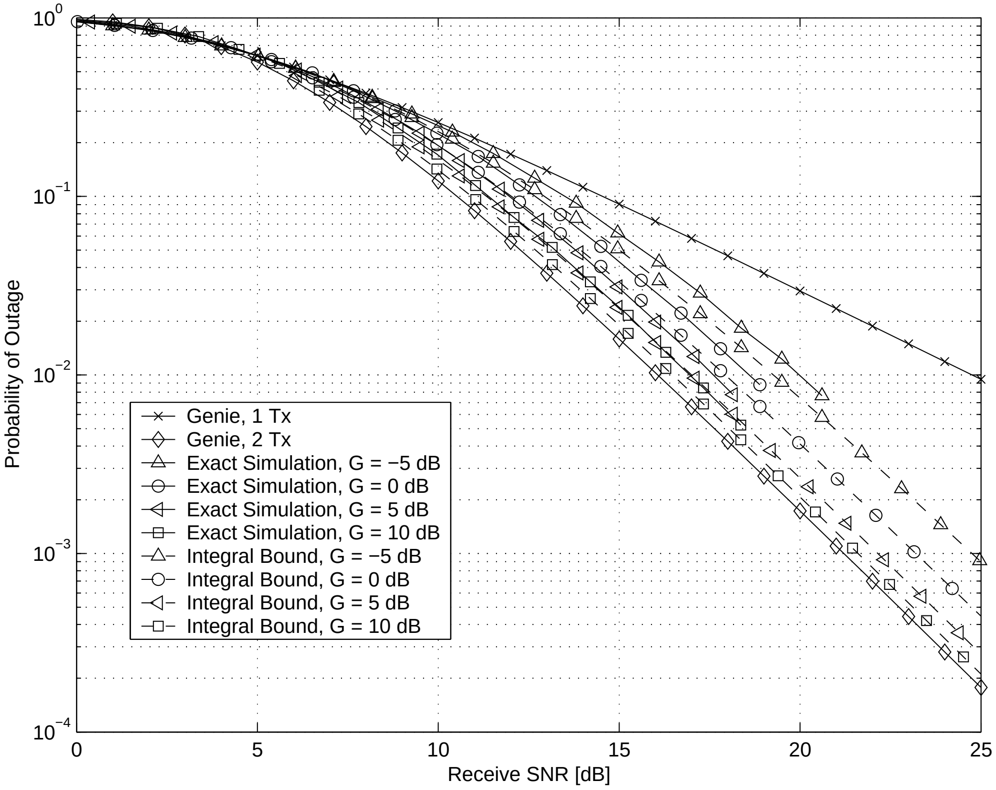
Simulation Results

2 Col, 1 Tx Each, 1 Rx, R = 0.5



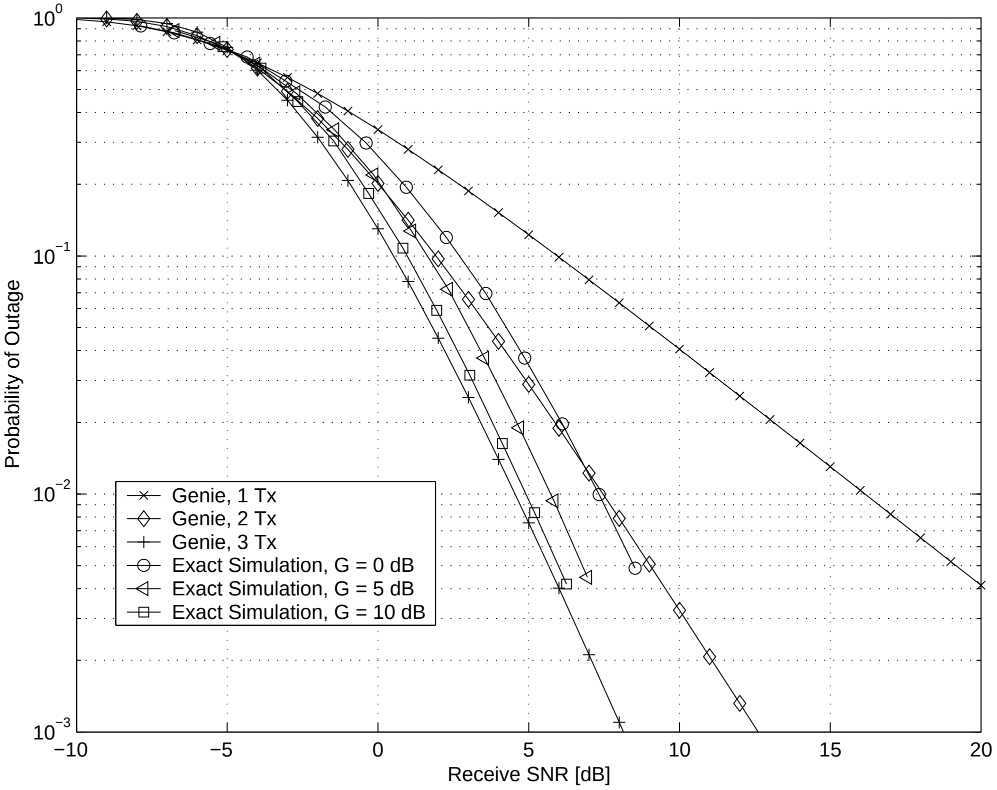
Simulation Results

2 Col, 1 Tx Each, 1 Rx, R = 2



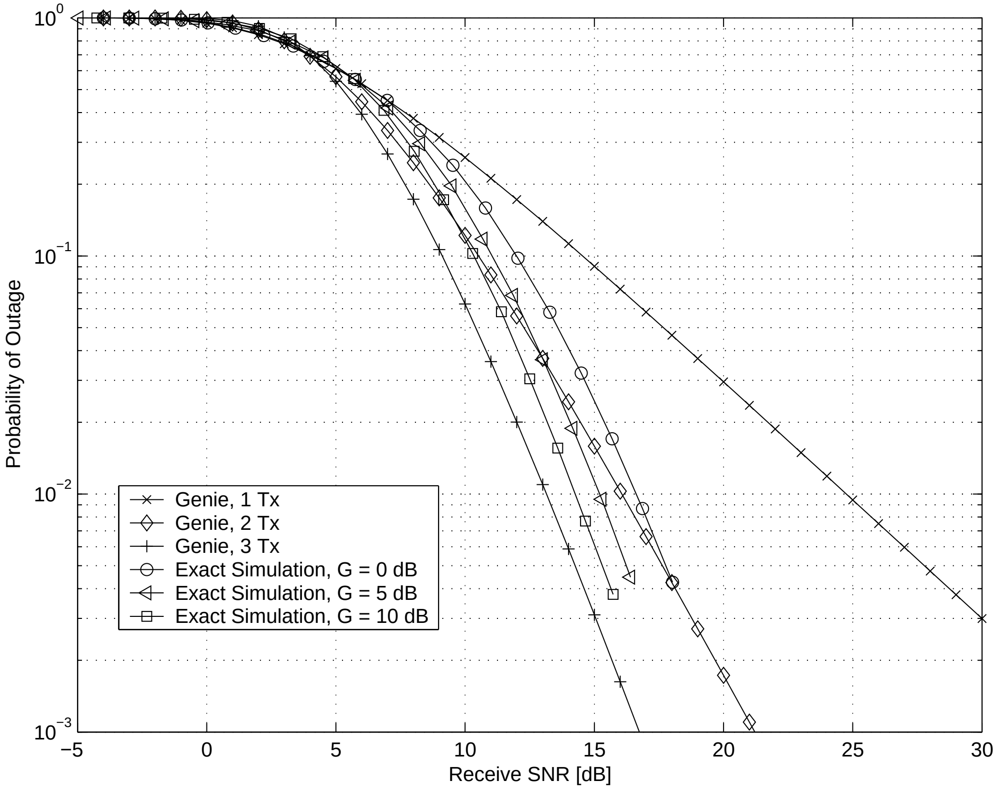
Simulation Results

3 Col, 1 Tx Each, 1 Rx, R = 0.5



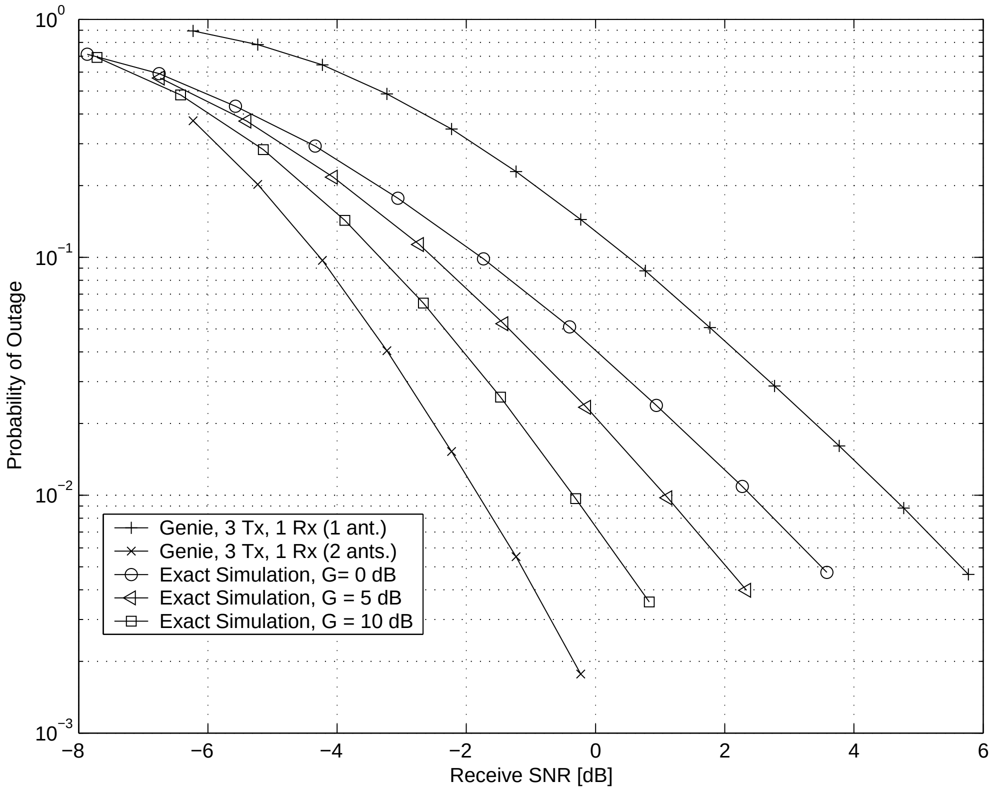
Simulation Results

3 Col, 1 Tx Each, 1 Rx, R = 2



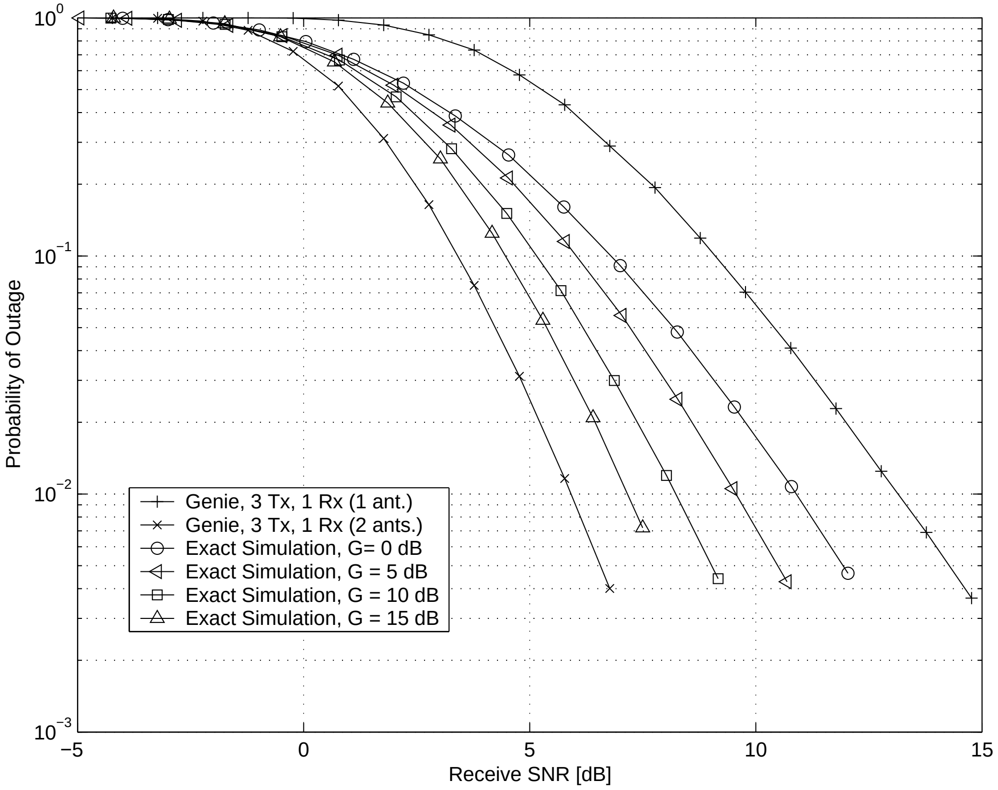
Simulation Results

3 Col, 1 Tx Each, 2 Rx, R = 0.5



Simulation Results

2 Col, 1 Tx Each, 2 Rx, R = 2



Conclusions

- We have considered a novel approach to compound relay channels and shown the existence of a collaborative code which is good over a wide range of relay channels.
- Our approach does not employ predetermined TDMA or orthogonal subchannels to resolve the half duplex constraint. Each relay, based on knowledge of its receive channel determines on its own when to listen and when to transmit.
- In addition to bandwidth efficiency, this approach has the advantage that each relay can receive information not only from the source, but via other relays. We have also show that the method may be applied to a bounded asynchronous scenario where each relay and destination node has an unknown integer delay with respect to the source.

- Numerical and simulation results have shown that if the intra-cluster communication has a 10 dB path loss advantage over the receiver at the destination node, in most cases there is essentially no penalty for the intra-cluster communication. Physically, in a two collaborator scenario, this corresponds to a transmit cluster whose radius is 1/3 the distance between the source and the destination.